

Charmonium

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Abstract

Topics in the description of the properties of charmonium states are reviewed with an emphasis on specific theoretical ideas and methods of relating those properties to the underlying theory of Quantum Chromodynamics.

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1 Introduction

Time is carrying us farther away from that day of November 11, 1974, when the news of an unusual resonance found simultaneously at BNL[1] and at SLAC[2] has swiftly spread through high-energy laboratories around the globe. Well before the age of widespread instantaneous satellite communications and the Internet, the new resonance became known in Moscow within the same hour as it was publicly announced in Palo Alto. After the initial excitement, confusion and revelations, it became clear that the new J/ψ resonance was the first to have been observed state of a system containing previously unknown (but anticipated) charmed quark and its antiquark: $c\bar{c}$ ¹. The new system, charmonium, in a close analogy with positronium or even with a hydrogen atom, was expected to contain a spectrum of resonances, corresponding to various excitations of the heavy quark pair. However, unlike its analogs governed mainly by the electrostatic Coulomb force, the properties of charmonium are determined by the strong interaction, so that the newly found system was, in a way, the simplest object for a study of the strong interactions. It was strongly hoped[4] that charmonium could play the same role for understanding hadronic dynamics as the hydrogen atom played in understanding the atomic physics. In a way, this has indeed been the case and the development of many methods in QCD is directly related to analyses of the properties of charmonium and of its heavier sibling bottomonium.

Recently the physics of charmonium regained a great renewed interest due to the massive dedicated investigation by BES and CLEO-c and the studies using decays of B mesons and the radiative return technique at the B factories with a higher initial energy of the electron and positron beams. After a ‘dry spell’ of more than two decades during which no new states of charmonium have been found with any certainty, new observations discover charmonium and charmonium-related resonances at a rate that outpaces the ability of the theory to fit their properties in a consistent scheme. Furthermore, the data very strongly suggest that among the new resonances there are exotic four-quark states, possibly hybrid states with gluonic degrees of freedom in addition to the $c\bar{c}$ pair, and also loosely bound states of heavy hadrons – charmonium molecules. Thus it looks like that charmonium not only has provided us with a ‘hadronic atomic physics’ but quite possibly also with a ‘hadronic chemistry’, and in its mature age of 33 charmonium still offers us new intriguing puzzles.

In this review some properties of the old and new states of charmonium and QCD-based methods of study of these properties are discussed. An emphasis is made on selected theoretical methods rather than on presenting the whole field and reviewing the data. An all-inclusive presentation can be found in the much larger review[5] and an excellent update on the most recent data and the related theoretical developments is given in Ref.[6].

The topics in the spectroscopy of the traditional charmonium states are provided in Section 2, and the annihilation of and the radiative transitions between these states are discussed in respectively Sections 3 and 4. The Section 5 is devoted to hadronic transitions between charmonium levels and the related topic of the interaction of slow charmonium with hadronic matter. Finally, in Section 6 are discussed peculiar properties of the resonances with masses above the open charm threshold.

2 The Spectrum of Charmonium States

2.1 General Considerations

The diagram of the known charmonium and apparently charmonium-related states is shown in Fig.1. The quantum numbers and basic properties of most of the states in the charmonium family can be described within a simple picture of a nonrelativistic quark - antiquark pair $c\bar{c}$. In this picture the states are characterized by the orbital angular momentum L , the total spin S of the quark pair, and

¹The second charmonium resonance was found just ten days after the J/ψ [3].

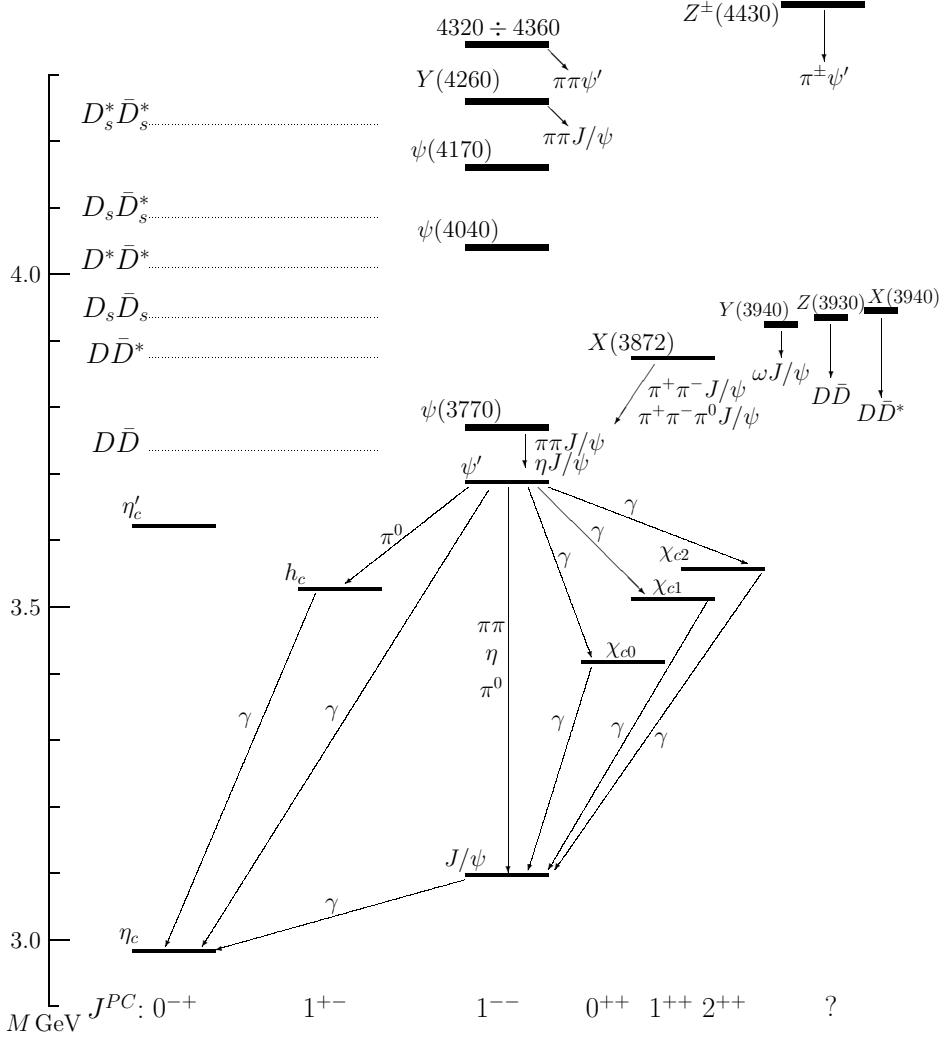


Figure 1: The known charmonium and charmonium-related resonances and some transitions between them. Also are shown (dotted lines) the thresholds for various pairs of charmed mesons.

the total angular momentum J , which defines the spin of the state viewed as a particle. As usual, the total angular momentum is given by the vector sum of the orbital and the spin momenta: $\vec{J} = \vec{L} + \vec{S}$. Likewise, the total spin S is determined by the vector sum of the quark and antiquark spins: $\vec{S} = \vec{s}_c + \vec{s}_{\bar{c}}$. Clearly, S takes the values 0 and 1, thus splitting the four possible spin states of the pair into a singlet and a triplet. Furthermore, the excitation of the radial motion of the $c\bar{c}$ pair results in a spectrum of levels with the same L , S and J , and differing by the “radial” quantum excitation number n_r with $n_r = 0$ corresponding to the lowest state in this spectrum. It is therefore customary to encode the values of these quantum numbers for each state of charmonium in the form of the symbol $(n_r + 1)^{(2S+1)} L_J$. The combination $2S + 1$ conveniently indicates the spin multiplicity, while following the tradition from atomic physics the values of L , $L = 0, 1, 2, 3, \dots$ are written as S, P, D, F, \dots . In this notation the lowest state with $L = 0, S = 0$ and (necessarily) $J = 0$ is represented as 1^1S_0 (η_c resonance) while the first excited state with the same quantum numbers is 2^1S_0 (η'_c).

The value of L determines the parity (P) for each of the states: $P = (-1)^{L+1}$, while L and S com-

bined also determine the charge conjugation parity: $C = (-1)^{L+S}$. Therefore the previously mentioned 1S_0 states have quantum numbers $J^{PC} = 0^{-+}$, while e.g. 3S_1 states have the same quantum numbers $J^{PC} = 1^{--}$ as the electromagnetic current, so that these states (J/ψ , ψ' , ...) can be produced as resonances in e^+e^- annihilation.

This briefly described standard nomenclature of the quark-antiquark states originates in the strictly nonrelativistic mechanics. Relativistic effects in the dynamics of quark and antiquark interacting with each other preserve most, but not all of it. Indeed, the conservation of the total angular momentum ensures that the states have definite J . On the contrary, the value of L is generally not preserved by the interaction. In particular, the operator of the so-called tensor forces $S(S+1) - 3(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{r})/r^2$ does not commute with \vec{L}^2 for the states with $S = 1$, so that a mixing of states with different L takes place. However the parity conservation requires that only the states with the same parity of L can get mixed, and the conservation of J then implies that the values of L for the mixed states can differ by at most two units. For instance a 3S_1 state can receive, due to relativistic effects and admixture of a 3D_1 state (' $S - D$ mixing'). It can be further noticed that the mixing in L is necessarily absent for certain states. Indeed all states with $J = L$ are pure in L , both the spin-singlets, $S = 0$, and the spin-triplets, $S = 1$, as are the 3P_0 resonances.

The applicability, in a certain extent, of a nonrelativistic description to charmonium has always been a source of interest to this system with the hope that in it the quark dynamics can be studied being not overly complicated by the relativistic effects. The significance of such effects in charmonium can be very approximately estimated already from the masses of the resonances, e.g. the mass difference ΔM between the ground 1S_0 state (J/ψ resonance) and its first radial excitation 2S_1 (ψ') in units of either of the masses provides an estimate of the relativistic parameter v^2/c^2 :

$$\frac{v^2}{c^2} \sim \frac{\Delta M}{M} \sim 0.2 . \quad (1)$$

Such moderate, but not very small magnitude of the relativistic effects in charmonium in fact makes some of these effects visible in experiments and thus further expands the range of dynamical details that can be studied in the charmonium system.

2.2 Potential Models

2.2.1 Leading Nonrelativistic Approximation

One widely used approach to describing charmonium is to consider its dynamics in analogy with atomic systems or positronium and to treat it in the nonrelativistic limit by means of a Schrödinger equation with a potential $V(r)$ depending on the distance r between the quark and the antiquark. The relativistic effects up to the order v^2/c^2 can then be considered as perturbation due to relativistic terms in the potential as well as in the kinetic energy. The shape of the potential at short distances is determined by the perturbation theory in QCD. In the lowest order the exchange of (Coulomb) gluons between slow quarks is fully analogous to interaction in QED, so that for a color-singlet quark pair the interaction takes the Coulomb-like form:

$$V_0(r) = -\frac{4}{3} \frac{\alpha_s}{r} , \quad (2)$$

where α_s is the QCD coupling. Once the scale dependence of this coupling is taken into account, the constant α_s in Eq. (2) has to be replaced by the running coupling $\alpha_s(r)$. At distances longer than the charmed quark Compton wave length $1/m_c$, the one-loop running is described by

$$\alpha_s(r) = \frac{2\pi}{9 \ln \frac{1}{r \Lambda_{QCD}}} . \quad (3)$$

In higher orders of the QCD loop expansion the precise relation of this ‘Coulomb-like’ coupling to the running constant defined in a specific renormalization scheme, such as e.g. the \overline{MS} scheme, is a matter of calculations, which have been carried to the two-loop level[7, 8] with some partial results[9] in three loops.

The details of these fine calculations however are not of an immediate significance for charmonium. The reason is that the perturbative QCD is applicable only at short distances, which are much shorter than the typical spatial size of the charmonium states. At the relevant intermediate and long distances one has to resort to models for the interaction between quarks. Some guidance in constructing such models is provided by the general idea of quark confinement, which can be mimicked by a potential rising at long distances. The most popular choice of the confining behavior is a linearly growing potential: $V(r) = \sigma r$. Such behavior originates in the idea of the contraction of the chromoelectric field between the quarks into a flux tube, giving a string-like binding.

The interaction potential can also be studied as the energy of a static infinitely heavy quark - antiquark pair separated by the distance r . This quantity can be evaluated in terms of the Wilson loop[10] by lattice QCD calculations. The numerical results of such analysis[11] produce a dependence of the static energy on r , which is in agreement with the Coulomb-like behavior at short distances and an approximately linearly rising potential at larger r .

It should be noted however that unlike in QED, a potential approach to heavy quarkonium in QCD is formally justified only in the limit of very high quark mass: in tens to hundreds GeV. In this limit the low-lying bound states in the short-distance potential (2) are localized at short distances, where the perturbative potential description is applicable and the whole approach is thus selfconsistent. Once nonperturbative effects in QCD are taken into account such consistency becomes questionable. Already the leading corrections[12, 13, 14] to energies of the quarkonium levels in the limit of very heavy quarks do not correspond to *any* potential between the quark and the antiquark. The reason for such behavior can be readily understood[12]. Indeed, a potential implies an instantaneous interaction. Any non-locality of the interaction in time would contain characteristic time scales, that can be interpreted as the evolution time scales for additional degrees of freedom. Once such additional degrees of freedom come into play the system (quarkonium) can no longer be described by a potential, neither it can be considered at all as a two-body system. In reality an interaction between the quark and antiquark through a gluon field should necessarily invoke nonperturbative light degrees of freedom in QCD whose typical evolution scale is determined by the QCD infrared parameter Λ_{QCD} . The interaction through exchange of such field can thus be viewed as instantaneous inasmuch as the quark and antiquark are slow in this scale, i.e. the characteristic time of evolution of the quarkonium wave function is long in this scale. For charmonium one can estimate the time of evolution as an inverse of the typical energy spacing between the levels, e.g. $M(\psi') - M(J/\psi) \approx 590$ MeV, which by any measure is certainly comparable with the QCD scale.

In other words, there in fact is no parameter that would justify a QCD-derived description of charmonium or bottomonium as a two-body quark-antiquark system interacting through a potential. However due to some numerical reasons, which are yet to be understood, such simple picture works reasonably well, especially if it is not pushed to requirements of high accuracy, or to highly excited states of quarkonium. It is almost so that any smooth potential whose behavior resembles Coulomb at short distances and an approximately linear rise at large r reasonably well describes the properties of the observed charmonium resonances, after the parameters of the model are appropriately adjusted. Some of the models for the potential discussed in the literature can be found in Refs. [15, 16, 17].

One of the most developed is the Cornell model[15, 18, 19] which builds upon the simplest potential being just a sum of the Coulomb and linear parts,

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a}, \quad (4)$$

and adding then finer effects, such as the relativistic terms, resulting in the hyperfine and fine structures

of charmonium levels, and also, importantly, including the coupling of the $c\bar{c}$ system to pairs of charmed mesons, such as $D\bar{D}$. The latter coupling effectively accounts for the fact that above the open charm threshold the charmed quarks and antiquarks do emerge inside the charmed hadrons, which effect is totally ignored in a pure potential model with confining interaction.

Clearly, a leading nonrelativistic treatment can only describe gross features of the charmonium levels, i.e. without resolving the fine splitting between the states with the same L and S and different J and the hyperfine splitting between the spin-triplet and spin-singlet states. It can be noted that even at such approximate level of detail the resulting sequencing of the energies of the levels with different n_r and L can provide useful constraints on the properties of the potential $V(r)$ [20].

2.2.2 Spin-dependent Forces

The potential description extended to spin-dependent interactions results in three types of interaction terms that are to be added to the discussed leading nonrelativistic interaction:

$$V_1(r) = V_{LS}(r) (\vec{L} \cdot \vec{S}) + V_T(r) \left[S(S+1) - \frac{3(\vec{S} \cdot \vec{r})(\vec{S} \cdot \vec{r})}{r^2} \right] + V_{SS}(r) \left[S(S+1) - \frac{3}{2} \right]. \quad (5)$$

The spin-orbit, V_{LS} , and the tensor, V_T , terms describe the fine structure of the states, while the spin-spin term, V_{SS} , proportional to $2(\vec{s}_q \cdot \vec{s}_{\bar{q}}) = S(S+1) - 3/2$ gives the spin-singlet - triplet splittings. The interaction in Eq.(5) arises among the v^2/c^2 effects in the nonrelativistic expansion and it generally requires additional model-dependent assumptions about the structure of the interquark forces. Within the phenomenological approach it is usually assumed that parts of the static potential (similar to that in Eq.(4)) correspond to definite Lorentz structures of the relativistic interaction between the quarks[21, 22, 4]. In other words, those structures correspond to an “exchange of something” with a definite spin between the quark and the antiquark. Then the short-distance Coulomb-like part of the static potential is naturally generalized as a limit of a vector type exchange:

$$(\bar{u}\gamma^\mu u)(\bar{v}\gamma_\mu v) V_V(q^2)$$

with u and v being the Dirac spinors for the quark and the antiquark, while the confining part has been treated in the literature as a part of a vector exchange[21, 22], or a scalar[5, 6], or a mixture of these[23]. With this choice of options restricted to a combination of only vector and scalar exchange, the spin-dependent terms in Eq.(5) can be written in terms of the vector, $V_V(r)$, and scalar, $V_S(r)$, parts of the static potential by the standard Breit-Fermi expansion to order v^2/c^2 [24]:

$$V_{LS} = \frac{1}{2m_c^2 r} \left(3 \frac{dV_V}{dr} - \frac{dV_S}{dr} \right), \quad (6)$$

where m is the charmed quark mass,

$$V_T = \frac{1}{6m_c^2} \left(\frac{d^2V_V}{dr^2} - \frac{1}{r} \frac{dV_V}{dr} \right), \quad (7)$$

and

$$V_{SS} = \frac{1}{3m_c^2} \Delta V_V, \quad (8)$$

with $\Delta = \nabla^2$ being the three-dimensional Laplacian.

It should be emphasized that by its nature the interaction in Eq.(5) is a part of the v^2/c^2 term in the nonrelativistic expansion. As such it can be used only in the first order, and by no means one should iterate this potential, or use it for an input in the Schrödinger equation. Therefore any accuracy of the

results found with potential of this type is intrinsically limited. Furthermore, the formulas (6) - (8) are only approximate even in perturbative QCD at short distances. In particular, the tree-level QCD potential (2), results, according to Eq.(8), in a point-like spin-spin interaction:

$$V_{SS} = \frac{16\pi\alpha_s}{9m^2} \delta^{(3)}(\vec{r}) , \quad (9)$$

which is the correct tree-level expression for the corresponding interaction in QCD. If one then improves the static potential by using the running coupling from Eq.(3), the formula (8) would produce terms extending to finite r and behaving as α_s^2/r^3 . A real calculation[25] of the one-loop correction to the spin-spin interaction, however produces no such terms, and predicts that the hyperfine splitting in perturbative QCD is still proportional to the square of the wave function at the origin, $|\psi(0)|^2$.

Clearly, the point-like behavior of the spin-spin interaction can generally be invalidated in higher orders in perturbative QCD and also by nonperturbative dynamics. Moreover, the leading nonperturbative effects in the limit of asymptotically heavy quarkonium[26] are not reduced to a point-like spin-spin interaction. Nevertheless, inspite of these reservations, the actual hyperfine splitting in charmonium closely resembles that produced by a short-distance interaction. Namely, the proportionality of the hyperfine splitting to $|\psi(0)|^2$ implies that the mass gap between the 3S_1 and 1S_0 states should be proportional to the e^+e^- decay width of the vector 3S_1 resonance, while the hyperfine splitting in the P wave should be extremely small because of vanishing wave function at the origin. In phenomenological terms this implies the following relations:

$$\frac{M(\psi') - M(\eta'_c)}{M(J/\psi) - M(\eta_c)} \approx \frac{\Gamma_{ee}(\psi')}{\Gamma_{ee}(J/\psi)} . \quad (10)$$

and

$$M(h_c) \approx \overline{M}(\chi_{cJ}) . \quad (11)$$

where $\overline{M}(\chi_{cJ}) = [5M(\chi_{c2}) + 3M(\chi_{c1}) + M(\chi_{c2})]/9$ is the ‘center of gravity’ of the 3P_J states which is not shifted by either the spin-orbital or the tensor interactions from Eq.(5). According to the Tables[27] the ratio of the e^+e^- decay rates in the r.h.s. of Eq.(10) is 0.45 ± 0.02 , while the ratio of the mass splittings in the l.h.s. is 0.44 ± 0.04 . Furthermore, the center of gravity of the spin-triplet χ_{cJ} states is at $\overline{M}(\chi_{cJ}) = 3525.36 \pm 0.06$ MeV. The spin-singlet 1P_1 state, the h_c , has been sighted[28] as a resonance in the $p\bar{p}$ annihilation with the mass $3526.28 \pm 0.18 \pm 0.19$ MeV, then in the same process[29] at the mass $3525.8 \pm 0.2 \pm 0.2$ MeV, and eventually[30] in the decays $\psi' \rightarrow \pi^0 h_c$ at $3524 \pm 0.6 \pm 0.4$ MeV, with the most recent improvement in the precision yielding[31] the h_c mass of $3525.35 \pm 0.19 \pm 0.15$ MeV. The data with the smallest claimed experimental errors point at an extremely small violation, if any, of the relation (11): $M(h_c) - \overline{M}(\chi_{cJ}) = -0.05 \pm 0.19 \pm 0.16$ MeV[31]. Thus the simple relations (10) and (11) both hold amazingly well. In fact it is still a challenge for experiment to measure the violation of these relations, and even a greater challenge to correctly predict such violation theoretically.

The spin-orbit and tensor terms in Eq.(5) produce the fine structure of the charmonium levels, and the tensor term also gives rise to mixing of states with L differing by two units, such as ${}^3S_1 - {}^3D_1$ mixing. The phenomenological effects of the mixing are somewhat more subtle and will be discussed further in Sec.3.1.2. Here we concentrate on the fine structure of the 3P_J states χ_{cJ} . The shifts of the masses of the states with different J with respect to $\overline{M}(\chi_{cJ})$ are given in terms of the averages $\langle V_{LS} \rangle$ and $\langle V_T \rangle$ over the P wave coordinate wave function as

$$\delta M({}^3P_0) = -2\langle V_{LS} \rangle + 2\langle V_T \rangle , \quad \delta M({}^3P_1) = -\langle V_{LS} \rangle - \langle V_T \rangle , \quad \delta M({}^3P_2) = \langle V_{LS} \rangle + \frac{1}{5}\langle V_T \rangle . \quad (12)$$

Using the measured differences between the masses of the χ_{cJ} charmonium resonances[27] one can find the average values over the $1P$ charmonium:

$$\langle V_{LS} \rangle = \frac{1}{12} [5M(\chi_{c2}) - 3M(\chi_{c1}) - 2M(\chi_{c0})] \approx 35 \text{ MeV} ,$$

$$\langle V_T \rangle = \frac{5}{36} [M(\chi_{c2}) - 3M(\chi_{c1}) + 2M(\chi_{c0})] \approx -20 \text{ MeV}. \quad (13)$$

This estimate illustrates that in order to describe the observed mass splitting between the χ_{cJ} states both the LS and tensor interactions are required with comparable strength. Furthermore, it can be noted that these forces, generated by a pure tree-level gluon exchange, as can be found from using the Coulomb-like potential V_V in the formulas (6) and (7), have correct signs, in agreement with the estimate (13), but with a relative strength of the LS interaction enhanced in comparison with this estimate: $V_{LS}/V_T|_{Coul} = -3$. Thus a certain reduction in the spin-orbit term due to a contribution of the Lorentz scalar potential V_S in Eq.(6) is indeed helpful from this point of view.

2.2.3 Potential Models and Predictions for New States

Once the parameters of a specific potential model are fixed from the data on the known states of charmonium, it is natural to use the same approach for predicting the masses of yet unobserved resonances corresponding to higher energy levels in the system. A large number of such predictions can be found in the literature spanning last three decades. Some of the results can be found in the papers [15, 18, 19, 23, 32, 33, 34] to name a few. The predictions for masses of the excited states considerably differ between models, which is not surprising in view of the nature of the approach. Moreover, given that the considered models do not have a controllable accuracy, it would be troublesome to assess what deviation of a prediction from the actual value of the mass should be regarded as successful.

Nevertheless, such application of the models appears far from being an entirely empty exercise, and in a way provides some gross features of the expected spectrum of higher excitations. Namely all the ‘reasonable’ models predict the same sequencing of levels: $M(1D) < M(2P) < M(3S)$, which can possibly be traced to the general properties[20], and the specific masses being generally in the following ranges: $M(1D) \approx 3.8\text{--}3.9 \text{ GeV}$, $M(2P) \approx 3.9\text{--}4.0 \text{ GeV}$, and $M(3S) > 4.0 \text{ GeV}$. The fine and hyperfine splittings of the levels are smaller than the uncertainty in the overall positions of the levels. All these states are above the $D\bar{D}$ threshold, so that most of them are expected to be broad due to decay into pairs of D mesons. The exception from this behavior can be found in the 3D_2 (2^{--}) and 1D_2 (2^{-+}) resonances if they are below the $D\bar{D}^*$ threshold at 3872 MeV. Indeed the unnatural spin-parity of these resonances forbids them to decay in the $D\bar{D}$ pairs, the only kinematically allowed states with open charm at such mass. The $2P_1$ and 3^1S_0 states are expected to be well above the $D\bar{D}^*$ threshold and for them such argument does not work.

The issue of higher excitation of charmonium has recently gained a great attention due to observation of a whole ‘zoo’ of new charmonium-like states in experiment. In particular those, which seem to relatively well fit the expected pattern of the levels are the resonances $Z(3930)$ and $Y(3940)$. The former state is found by Belle[35] in $\gamma\gamma$ production with the mass and width $M(Z) = 3929 \pm 5 \pm 2 \text{ MeV}$ and $\Gamma(Z) = 29 \pm 10 \pm 2 \text{ MeV}$ decaying mostly to $D\bar{D}$, and which reasonably fits[19] the slot for the 2^3P_2 (χ'_{c2}) state of charmonium. The latter resonance, $Y(3940)$, observed by Belle[36] in the $\omega J/\psi$ channel in the decays $B \rightarrow \omega J/\psi K$ has the parameters $M(Y) = 3943 \pm 11 \pm 13 \text{ MeV}$ and $\Gamma(Y) = 87 \pm 22 \pm 26 \text{ MeV}$. It appears to not decay into the pairs of pseudoscalar mesons $D\bar{D}$ and can be considered[39] as a candidate for the 2^3P_1 (χ'_{c1}) state of charmonium. If this interpretation proves to be correct an interesting spectroscopic question would arise related to the apparently inverted or small mass splitting between the 2^3P_1 and 3^3P_2 states. Most recently the peak in the invariant mass of the system $\omega J/\psi$ possibly consistent with $Y(3940)$ was observed by BaBar[37] in the decays $B^+ \rightarrow \omega J/\psi K^+$ and $B^0 \rightarrow \omega J/\psi K_S$. The values for the mass and width of the observed peak are somewhat off compared to the initial observation: $M(Y) = 3914^{+3.8}_{-3.4} \pm 1.9 \text{ MeV}$ and $\Gamma(Y) = 33^{+12}_{-8} \pm 5 \text{ MeV}$. Thus the status of this resonance is still not clear. The suggested interpretations include an excited P wave quarkonium[6], hybrid $c\bar{c}g$ state[38], and a four-quark molecular state.

Another recently found resonance $X(3940)$ is observed[40] as recoiling against J/ψ in $e^+e^- \rightarrow$

$J/\psi + X$, which implies that its C parity is positive. Furthermore it appears to decay into $D\bar{D}^*$ but not into $D\bar{D}$, so that it likely has unnatural spin-parity. These properties invite an interpretation[19, 39] of the resonance as a 0^{-+} charmonium state which would then be $\eta_c(3S)$. A possible problem of such interpretation is that most of the models expect the $3S$ level in charmonium to be somewhat above 4.0 GeV, so that if further study of $X(3940)$ indeed identifies it as a 0^{-+} resonance, this may bring some new interesting understanding of hadron dynamics.

2.3 Spectral Methods

The potential models of heavy quarkonia and of charmonium in particular, are intuitively appealing, versatile and very convenient for estimates of various characteristics of the heavy resonances, but they can not be entirely satisfactory due to their model-dependent relation to the underlying theory of QCD. More directly related to the first principles of QCD are the methods based on the spectral relations for correlators in QCD. Such approach can be illustrated in its most basic form by considering the correlation function of the type $F(x) = \langle 0 | T \{ O^\dagger(x), O(0) \} | 0 \rangle$, where $O(x)$ is a local operator and $|0\rangle$ is the vacuum state in QCD. Of relevance to charmonium is the choice of the operator $O(x)$ where it contains a factor $\bar{c}\Gamma c$ (with some structure Γ) and thus produces states of charmonium. The correlation function can be written in terms of the spectral sum over the physical states $|n\rangle$ containing a $c\bar{c}$ pair:

$$F(x) = \sum_n |\langle n | O | 0 \rangle|^2 D_n(x) , \quad (14)$$

where $D_n(x)$ is the propagator of the state n . The lowest mass states contributing to the sum are the one-particle states i.e. the charmonium resonances, while at higher mass the sum is also contributed by the continuum of states containing the charm - anticharm quark pair. By an appropriate choice of the operator $O(x)$ one projects out the states with particular quantum numbers, while a suitable choice of x allows to make the sum being dominated by the charmonium resonances of interest. The direct relation to ‘the first principles’ arises when the correlator F can be also calculated in the interesting range of x by methods of the underlying QCD theory, thereby relating the phenomenological properties of hadrons to the results of a QCD calculation.

The two approaches to calculating the correlator F are the numerical calculations in lattice QCD and the short-distance QCD analytical treatment. The lattice approach in principle allows to evaluate the correlator at large Euclidean separation x where the spectral sum is given by only the lowest mass state, so that e.g. the mass of this state can be fully determined. On the other hand, the short-distance QCD methods are restricted to relatively small values of the interval x , so that the spectral sum still contains some contribution from higher states as well as the lowest one. For this reason the relations resulting from calculations of this type are known as the QCD sum rules. The usefulness of the sum rules for description of the lowest state in a given channel depends on the existence of an intermediate range of a parameter analogous to x , where both the theoretical uncertainty in a short-distance calculation and the phenomenological uncertainty of the contribution of the higher mass states to the spectral sum can be reasonably controlled.

2.3.1 QCD Sum Rules

It is due to the e^+e^- annihilation data that the most well studied channel with hidden charm is the vector one, i.e. corresponding to the charm - anticharm production by the electromagnetic current of the charmed quarks $j_\mu = (\bar{c}\gamma_\mu c)$, and this is the channel for which the original QCD sum rules were developed[41]. The relevant correlator is then the vacuum polarization $P(q^2)$, considered in the momentum, rather than the position space:

$$P(q^2) \left(-q^2 g_{\mu\nu} + q_\mu q_\nu \right) = i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu(x), j_\nu(0) \} | 0 \rangle . \quad (15)$$

The spectral sum then takes the form of the dispersion relation for $P(q^2)$,

$$P(q^2) = \frac{q^2}{\pi} \int \frac{\text{Im}P(s)}{s(s - q^2 - i\epsilon)} ds , \quad (16)$$

and the imaginary part $\text{Im}P(s)$ is related to the contribution of the states with hidden charm $R_c(s)$ to the measured cross section ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/(4\pi\alpha^2/3s)$:

$$\text{Im}P(s) = \frac{R_c(s)}{12\pi} . \quad (17)$$

At values of q^2 sufficiently below the (perturbative) threshold at $4m_c^2$ the vacuum polarization $P(q^2)$ is determined by the QCD dynamics at short distances and can be calculated by using the Operator Product Expansion (OPE) for the T product in Eq.(15),

$$T \{j(x), j(0)\} = \sum_d c_d(x) \mathcal{O}_d(0) \quad (18)$$

in terms of local operators $\mathcal{O}_d(0)$ with increasing dimension d , and $c_d(x)$ being the coefficient functions calculable in QCD. The leading operator of lowest dimension in this expansion is the unit operator \mathcal{I} , and the corresponding coefficient $c_0(x)$ includes all the QCD perturbation theory result for $P(q^2)$. The first nontrivial operator in the series is the next one with dimension $d = 4$ and is quadratic in the gluon field strength tensor $G_{\mu\nu}^a$, so that its contribution to the vacuum polarization is proportional to the gluon vacuum condensate[42, 43, 4, 44]. The relation between the theoretical expression for $P(q^2)$ and the phenomenological integral over the observed cross section can be studied as a function of q^2 far below the threshold. Alternatively, one can compare the expressions for the derivatives of the vacuum polarization with respect to q^2 at $q^2 = 0$, for which the ‘phenomenological’ side of the sum rules is given by the moments of the ratio R_c :

$$\mathcal{M}_n = \int \frac{R_c(s)}{s^{n+1}} ds , \quad n = 1, 2, \dots , \quad (19)$$

$\mathcal{M}_n = (12\pi^2/n!) (d^n P(z)/dz^n)|_{z=q^2=0}$. Theoretically, the Taylor expansion of the vacuum polarization including the leading nonperturbative term can be written as

$$P(q^2) = \sum_{n=1} \left(\mathcal{C}_n + \mathcal{D}_n \frac{\langle 0|G^2|0\rangle}{m_c^4} \right) \left(\frac{q^2}{4m_c^2} \right)^n , \quad (20)$$

where \mathcal{C}_n and \mathcal{D}_n are dimensionless coefficients that are calculated as series in powers of α_s . The coefficients \mathcal{C}_n are known[45] at arbitrary n up to α_s^2 , and \mathcal{C}_1 has been recently evaluated[46, 47] to order α_s^3 . The coefficients \mathcal{D}_n are known in the lowest[42, 4] and the next to lowest[48] orders in α_s .

The contribution of a narrow resonance to R_c can be approximated by a δ function:

$$R_c(s) = \frac{9\pi}{\alpha^2} \delta(s - M^2) \Gamma_{ee} M , \quad (21)$$

where M is the mass of the resonance and Γ_{ee} is the width of its decay into e^+e^- . The lowest charmonium state contributing to the dispersion integral in Eq.(16) is the J/ψ resonance, so that the n -th moment can be written as

$$\mathcal{M}_n = \frac{9\pi}{\alpha^2} \left[\frac{\Gamma_{ee}(J/\psi)}{M^{2n+1}(J/\psi)} + \frac{\Gamma_{ee}(\psi')}{M^{2n+1}(\psi')} \right] + \int_{s > M^2(\psi')} \frac{R_c(s)}{s^{n+1}} ds , \quad (22)$$

where the latter integral runs over c.m. energies above the ψ' resonance. One can readily see that the relative weight of the lowest resonance grows with the number of the moment n . (E.g. the J/ψ

contribution essentially dominates the moments of charm cross section already at $n = 3 \div 4$.) However the correction terms, both the perturbative and nonperturbative, in the theoretical calculation of the moments also grow with n , which is a reflection of the fact that higher derivatives are sensitive to the threshold singularity in the correlator where the essential distances are no longer short. In particular the parameter for the perturbative expansion for the moments is effectively $\alpha_s \sqrt{n}$ (corresponding to the α_s/v behavior of effects of the Coulomb-like interaction near the threshold), and for the leading nonperturbative term due to the gluon condensate behaves as $n^3 \langle G^2 \rangle / m_c^4$. It was found nevertheless[41, 4] that the perturbative one-loop expression for the moments can be relied on up to $n \approx 4$, and somewhat higher moments (up to $n = 7 \div 8$) then can be used in order to determine[42, 43] the value of the gluon vacuum condensate $\langle G^2 \rangle$. Once other relevant QCD parameters (α_s , the charmed quark mass m_c) were determined from the data and the sum rules for the vector channel, they could be used for other channels as well. In particular, the sum rules for the spectral density of the pseudoscalar operator ($\bar{c}\gamma_5 c$) correctly predicted[42] the mass of the lowest 0^- charmonium resonance, the η_c .

The best theoretically known is the first moment of $R_c(s)$, due to the accuracy (α_s^3) of the available perturbative calculation and due to very small nonperturbative term proportional to $\langle G^2 \rangle$. On the experimental side this moment is rather sensitive to the details of the charm production cross section in e^+e^- annihilation at around 4.0 GeV where the open charm production sets in, and the energy behavior of the cross section is quite complicated. Recent data in this region [49, 50] have allowed to put to use the attained theoretical accuracy in M_1 . In recent analyses the sum rule for the first charm moment was used for precision determination of the short-distance charmed quark mass parameter, $\overline{m}_c(\overline{m}_c)$: 1295 ± 15 MeV[47] and 1286 ± 13 MeV[51].

2.3.2 Lattice Methods and Limitations of the Spectral Approach

The QCD sum rule approach, based on the OPE and analytical calculations is certainly limited by the applicability of the perturbative expansion in α_s . This requires a careful choice of parameters, e.g. the number of the moment n , to ensure such applicability and still get a phenomenologically useful relation. It is widely believed that a ticket to calculations beyond the perturbation theory in α_s that are not limited to short-distance QCD is the numerical lattice approach. In particular, a natural application of this approach would be a calculation of correlators at large separation between the points in the Euclidean space, so that the contribution of the lowest states in each channel could be completely determined. However practical implementations of lattice calculations to heavy quarkonium including charmonium still run into difficulties of their own. A detailed discussion of recent developments of the lattice methods and of the associated difficulties can be found in the review[5]. As of this writing the accuracy of the lattice results even for the masses of the lowest states of charmonium in each J^{PC} channel leaves an ample room for further improvement.

The spectral approach in general, based on analytical or numerical calculations, has ‘built in’ a certain deficiency with regards to excited states. Namely the spectral sum for a correlator at a Euclidean separation necessarily receives the largest contribution from the lowest physical state. By increasing the separation one can at best enhance the sensitivity to the properties of the lowest state and thereby evaluate those properties. There is however no simple way of ‘focusing’ spectral relations on excited states, so that a study of those states by spectral methods runs into additional difficulties.

Furthermore, even at the level of sum rules, or of a study of the lowest charmonium states in each channel by lattice methods, the spectral approach eventually runs into limits of its accuracy arising due to annihilation of charmonium into light hadrons. This limitation can also be viewed as a version of the problem of excited states: the lowest state of charmonium in a channel with given quantum numbers J^{PC} is certainly not the lowest hadronic state in that channel. In other words, at some level an operator ($\bar{c}\Gamma c$) produces states containing only light quarks and gluons and no charmed quarks. The old phenomenological rule, according to which a mixing of hidden quark flavors, although not forbidden

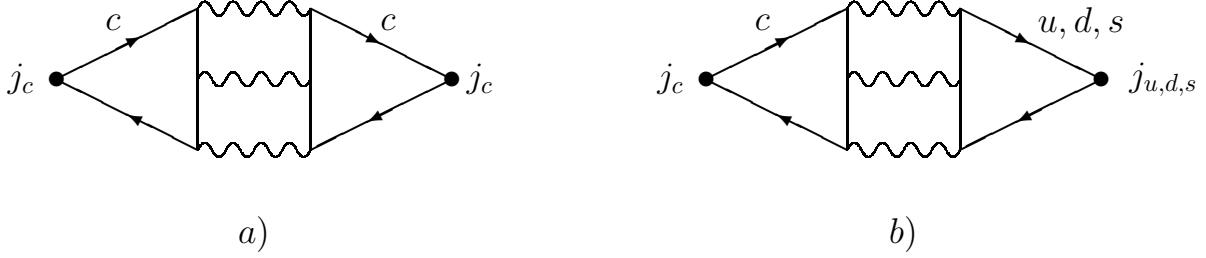


Figure 2: Representative graphs for OZI rule violation in the vacuum polarization by the electromagnetic current j_c of charmed quarks (a) and for the interference between the charmed quark current and that of the light quarks (b). The wavy lines show the gluons.

by any conservation laws, is dynamically suppressed, is traditionally referred to as the Okubo[52]-Zweig[53]-Iizuka[54] (OZI) rule, so that the discussed effect is precisely the violation of this rule. The significance of this effect generally depends on the channel considered, and as also will be mentioned in the Section 3.2.3, it can be essential in the properties of the η_c resonance. In the considered above case of the vector channel this effect is likely to be quite small, fully in line with the known strong suppression of OZI violation in vector mesons. Indeed, the partial width of the J/ψ resonance associated with strong annihilation of the hidden charm into light hadrons is only about 70 keV, which can be viewed as the imaginary part of the shift of the mass of the resonance due to the OZI violation. Generally the real part of the shift is of the same order as the imaginary, which for the J/ψ meson would be much smaller than any current accuracy of a theoretical calculation of its mass.

It can however be noted that the already achieved accuracy in the first moment of $R_c(s)$ is actually on the verge of being sensitive to the effects of the OZI rule violation. Indeed, the separation of the hidden charm from light degrees of freedom in the electromagnetic production is broken in order α_s^3 due to the mechanisms illustrated in Fig.2. The theoretical calculation of the first moment \mathcal{M}_1 happens to be insensitive[55, 56] to the contribution of the three-gluon intermediate states shown in Fig.2a, while the cross term between the currents of the light and charmed quarks, shown in Fig.2b does not enter the correlator in Eq.(15) by definition. The absence of the contribution from the graphs of the type shown in Fig.2a in \mathcal{M}_1 (as well as in \mathcal{M}_2 and \mathcal{M}_3) can be assured by the following reasoning. Consider this mechanism at small q^2 , so that $q^2 \ll m_c^2$. In this region the charmed quark loop can be contracted into a point thus generating an effective point-like Lagrangian for coupling of the current to three gluons. Similarly to the well-known Heisenberg-Euler Lagrangian for the photons, the current conservation and the QCD gauge invariance ensure that the mass of the fermion in the loop enters as $1/m_c^4$. The graph of Fig.2a contains two such heavy quark loops, so that the low-energy expansion of the contribution of the discussed mechanism to the correlator of the currents starts as $\alpha_s^3 (q^2/m_c^2)^4 \ln(q^2/m_c^2)$, and therefore no contribution to the first three moments \mathcal{M} results from this mechanism.

The cross-talk between the electromagnetic currents of the light and charmed quarks illustrated in Fig.2b in fact relates to the general problem of phenomenological separation of the hidden-charm and light-quark contribution both to the measured cross section and to the sources of the observed final states. The former separation of the hidden-charm part from the total cross section is usually done by subtracting from the data the smooth background associated with the light states, while the latter separation of the sources corresponds to evaluating how much of light states are produced by the current of charmed quarks and how much of the hidden charm is produced by the electromagnetic current of the light quarks. The cross-talk effect, however is suppressed by the flavor $SU(3)$ symmetry. Indeed, the electromagnetic current of the light quarks is a pure component of $SU(3)_{fl}$ octet, so that in the limit of

exact symmetry this current does not produce the heavy quarks, which are $SU(3)_{fl}$ singlets. This effect is further suppressed in the first moment \mathcal{M}_1 (in this case only in the first moment). The reasoning is similar to the one regarding Fig.2a. In this case there is only one heavy quark loop proportional to $1/m_c^4$. Thus the low-energy expansion of the total sum of the graphs of the type shown in Fig.2b necessarily starts with $(q^2/m_c^2)^2 \ln(q^2/m_c^2)$, giving no contribution to \mathcal{M}_1 .

It can be also mentioned that in the vector channel the mixing of light hadrons and the hidden charm starts in the order α_s^3 due to the C parity. In the C -even channels such mixing starts in the order α_s^2 (except for the $J = 1$ channels, where it is forbidden by the Landau-Yang theorem[57, 58, 4]), so that the OZI violating effects should be larger in the spectroscopy of these channels. Furthermore, the $J = 0$ channels 0^{-+} and 0^{++} generally suffer from large quarks-glue mixing effects, related to direct instantons[59], which may hold the clues to understanding some properties of the η_c and χ_0 resonances.

3 Charmonium Annihilation

The same OZI violating effects that are complicating an improvement in the precision of calculation of the masses of charmonium resonances are also the sole reason for the eventual disappearance of the hidden charm, i.e. for annihilation of the $c\bar{c}$ quark pair into light states. The two interactions contributing to the annihilation decays are the strong interaction responsible for most processes of this type, and the electromagnetic interaction which is relevant to annihilation decays of the vector 1^{--} states and the 2γ decay widths of the C even states with $J \neq 1$. We first consider in this section the electromagnetic processes.

3.1 Electromagnetic Annihilation

3.1.1 Annihilation of 3S_1 States through Virtual Photon

The 3S_1 states have quantum numbers of a virtual photon, $J^{PC} = 1^{--}$ and can annihilate into lepton pairs or light hadrons through one photon. This is also the process, which when reversed, gives rise to the formation of the 3S_1 states as resonances in e^+e^- annihilation. The rate of the decay can be estimated in the extreme-nonrelativistic picture, where the system is described by the wave function $\psi(\vec{r})$ for the quark-antiquark pair and depending on their relative position $\vec{r} = \vec{r}_c - \vec{r}_{\bar{c}}$. The annihilation takes place at the characteristic distances of order $1/m_c$ which are to be viewed as $r \rightarrow 0$ for a nonrelativistic pair, so that the annihilation amplitude is proportional to the wave function at the origin. For an n -th S -wave state the wave function can be written entirely in terms of its radial part R_{nS} : $\psi_{nS}(\vec{r}) = R_{nS}/\sqrt{4\pi}$, and the expression for the rate of annihilation into e^+e^- takes the form

$$\Gamma_{ee}(n{}^3S_1) = \frac{4\alpha^2 e_c^2}{M^2} |R_{nS}(0)|^2 \left(1 - \frac{16\alpha_s}{3\pi}\right), \quad (23)$$

where $e_c = 2/3$ is the electric charge of the charmed quark in units of the fundamental charge, M is the mass of charmonium, and finally, the term with α_s in the parenthesis gives the first QCD correction. The coupling constants α_s and α should be taken at the scale m_c (or M) as appropriate for a process proceeding at distances $\sim 1/m_c$. The correction (sometimes forgotten) due to the running of the QED coupling is reasonably tractable and results in approximately 7% enhancement of the rate, while the first QCD correction is quite discouraging. Indeed its numerical value, $-1.7\alpha_s$, amounts to $-(0.35 \div 0.5)$ for $\alpha_s(m_c)$ in the realistic range $0.2 \div 0.3$, which tells us that higher QCD corrections can be quite essential[60].

There are however other uncertainties of a conceptual nature associated with the formula (23). One source of such uncertainties arises from the use of the nonrelativistic (essentially static) approximation. One of the indications of this approximation is the parameter M : “the mass of charmonium” entering

Eq.(23). It would be impossible to specify at this level of approximation whether M should be set at twice the quark mass $2m_c$, or the mass of the specific decaying state, or some combination of those, since the difference between these values for M is formally of order v^2/c^2 . Clearly, the first relativistic correction can amount to tens percent. It should be noted that the specific form of this correction does not come from purely kinematical effects in the annihilation of moving, as opposed to static, quarks, but is also sensitive to the interaction, since the average kinetic energy is of the order of the average interaction potential as follows from the virial theorem. Another fundamental uncertainty of Eq.(23) is related to the assumption that the quarkonium can be described by a quark-antiquark two-body wave function, which assumption is formally not valid in nonperturbative QCD, as was discussed in Sec.2.2.1. The nonperturbative correction to the width Γ_{ee} of the 3S_1 states has been calculated[13] in the limit of very heavy quarkonium, where the correction is parametrically small. An extrapolation down to charmonium would produce an unreasonably large result[60], neither it would be justified. Thus the issue of the applicability of a description of charmonium as a two-body system arises again as previously in the general discussion of potential models. If in spite of this issue potential models are used in conjunction with Eq.(23) and similar formulas for other annihilation rates that will be discussed in this section, the results are in a qualitative agreement with the observed pattern of the decay rates, although no controllable accuracy can be assigned to such results.

The electromagnetic annihilation contributes a sizable fraction, about a quarter, of the total decay width of the J/ψ resonance. The latest precise data[61] have moved the world average[27] for the e^+e^- branching ratio to $\mathcal{B}_{ee}(J/\psi) = (5.94 \pm 0.06)\%$. The decay into $\mu^+\mu^-$ goes with essentially the same rate, and the rate of electromagnetic annihilation into light hadrons is given by the e^+e^- rate scaled by the ratio R measured in e^+e^- annihilation just below the J/ψ resonance[49]: $R(3.0\text{ GeV}) = 2.21 \pm 0.05 \pm 0.11$, thus giving the total branching fraction for the decays of J/ψ through a virtual photon as

$$\mathcal{B}(J/\psi \rightarrow \gamma^* \rightarrow X) = (2 + R) \mathcal{B}_{ee}(J/\psi) = (25.0 \pm 0.8)\%. \quad (24)$$

For the ψ' resonance the contribution of the electromagnetic annihilation is by far less prominent (partly due to larger total decay width), given that $\mathcal{B}_{ee}(\psi') = (7.35 \pm 0.18) \times 10^{-3}$ and using[49] $R(3.7\text{ GeV}) = 2.23 \pm 0.08 \pm 0.08$, one can estimate

$$\mathcal{B}(\psi' \rightarrow \gamma^* \rightarrow X) = (2 + 0.39 + R) \mathcal{B}_{ee}(\psi') = (3.39 \pm 0.11)\%, \quad (25)$$

where the term 0.39 in the parenthesis accounts for the decay $\psi' \rightarrow \tau^+\tau^-$. The theoretical ratio of the rate of this decay to $\Gamma_{ee}(\psi')$ is in agreement with the direct measurement[27], but has about ten times smaller error. In terms of potential models the ratio of the e^+e^- decay rates for ψ' and J/ψ can serve according to Eq.(23) as a measure of the ratio of the wave functions at the origin, and this estimate was used above in connection with the relation in Eq.(10).

The e^+e^- decay rates of the vector resonances appear directly in the spectral approach (Eq.(21)), so that the relativistic effects and perturbative and nonperturbative QCD terms are all included inasmuch as these effects are taken into account in calculation of the correlation function. However, as mentioned previously, the spectral relations are mostly sensitive to the contribution of the lowest resonance, i.e. the J/ψ in charmonium. It is known since long ago[41, 4] that the QCD sum rules are in an excellent agreement with the observed value of $\Gamma_{ee}(J/\psi)$ of about 5 keV.

3.1.2 Annihilation of mixed ${}^3D_1 - {}^3S_1$ States through Virtual Photon

The quantum numbers of a 3D_1 state are $J^{PC} = 1^{--}$ and allow such state to decay through one virtual photon. However in a pure D wave state the wave function at the origin is vanishing, so that in the leading nonrelativistic approximation the annihilation amplitude is zero. A non-vanishing contribution to this amplitude arises in the order v^2/c^2 due to two mechanisms. The first one is the ${}^3D_1 - {}^3S_1$ mixing

due to the tensor force, while the second arises from the expansion of the D wave coordinate wave function at the annihilation distances $1/m_c$ and is proportional to the second derivative of the radial function at the origin: $R''_{nD}(0)/m_c^2$, which is also of order $p^2/m^2 \sim v^2/c^2$ in comparison with the S -wave wave function at the origin. The latter mechanism alone would result in the expression for the width[4]

$$\Gamma_{ee}(n^3D_1) = \frac{200 \alpha^2 e_c^2}{M^6} |R''_{nD}(0)|^2 , \quad (26)$$

where no short-distance QCD correction is included. The direct annihilation amplitude and the one due to the mixing fully interfere with each other and are of the same order in v^2/c^2 . Thus it would generally be unjustified to consider one mechanism but not the other.

Phenomenologically, the resonance $\psi(3770)$ is considered to be such dominantly 1^3D_1 state with an admixture of 3S_1 wave function. The simplest model for the latter admixture uses the proximity in mass of the ψ' resonance and considers only the two state $\psi(3770) - \psi'$ mixture. Then, using the data[27] for $\Gamma_{ee}[\psi(3770)] = 0.242^{+0.027}_{-0.024}$ keV and ignoring the direct annihilation amplitude one can estimate[62, 63] the mixing angle θ as being about 0.2. Such estimate certainly agrees with the expectation for the size of the v^2/c^2 effects in charmonium, although the particular numerical value should likely be taken with certain reservations given the very simplistic nature of the model.

3.1.3 Two-photon Annihilation of C -even States

The $c\bar{c}$ quark pair in C even states with $J \neq 1$ can annihilate into two photons[64, 4]. For the n^1S_0 states the amplitude is proportional, in a potential model approach, to the wave function at the origin, $R_{nS}(0)$, so that it makes sense[65] to consider the ratio of the ${}^1S_0 \rightarrow 2\gamma$ and $J/\psi \rightarrow e^+e^-$ decay rates where the value of the wave function at the origin cancels. Including also the first short-distance QCD correction[66] for the 2γ decay, which can be traced back to the positronium result[67], one can write

$$\frac{\Gamma(n^1S_0 \rightarrow \gamma\gamma)}{\Gamma_{ee}(n^3S_1)} = 3 e_c^2 \left[1 + \frac{\alpha_s}{3\pi} (\pi^2 - 4) \right] = \frac{4}{3} \left(1 + 1.96 \frac{\alpha_s}{\pi} \right) . \quad (27)$$

Experimentally the 2γ decay rate is measured for the η_c , albeit with a large uncertainty[27], so that for the $1S$ charmonium the ratio of the rates in Eq.(27) is experimentally 1.3 ± 0.4 . Although this value can be considered as being in agreement with the theoretical expectation, measurements with smaller experimental uncertainty are clearly needed for a more meaningful comparison. It would also be of a great interest to test Eq.(27) for the $2S$ charmonium, i.e. for the corresponding decay rates of the ψ' and η'_c resonances. So far the process $2\gamma \rightarrow \eta'_c$ has been seen[68, 69] as a source of η'_c in $\gamma\gamma$ collisions, however quantitative data on decay widths of the η'_c are still in flux.

It can be mentioned that the rate of the decay $\eta_c \rightarrow 2\gamma$ can be analysed by considering the QCD sum rules for the moments of the $\gamma\gamma$ scattering cross section due to the electromagnetic current of the charmed quarks[70, 4]. The η_c resonance is the lowest state contributing to this cross section for perpendicular linear polarizations of the photons. The result of such estimate, not including QCD corrections and nonperturbative terms is[4] $\Gamma(\eta_c \rightarrow 2\gamma) = (6.1 \div 6.5)$ keV, which is also in agreement with the existing data[27].

The amplitudes of the two-photon annihilation of the P -wave states, 3P_0 and 3P_2 are proportional to the first derivative of the radial wave function at the origin: $R'_P(0)/m_c$. The specific expressions[64] with the first short-distance QCD correction[71] are given by

$$\begin{aligned} \Gamma({}^3P_0 \rightarrow \gamma\gamma) &= \frac{2^4 3^3 e_c^4 \alpha^2}{M^4} |R'_P(0)|^2 \left[1 + \frac{\alpha_s}{3\pi} \left(\pi^2 - \frac{28}{3} \right) \right] , \\ \Gamma({}^3P_2 \rightarrow \gamma\gamma) &= \frac{2^6 3^2 e_c^4 \alpha^2}{5 M^4} |R'_P(0)|^2 \left(1 - \frac{16 \alpha_s}{3\pi} \right) . \end{aligned} \quad (28)$$

Both of these rates are of order v^2/c^2 in comparison with the similar rate for the 1S_0 state. Thus one should expect a certain suppression of these decays of the χ_{cJ} resonances as compared to the rate $\Gamma(\eta_c \rightarrow \gamma\gamma)$, although an absolute prediction of the rates would be quite model dependent due to uncertainty in the value of the wave function (as well as due to other uncertain factors discussed in connection with Eq.(23)). The experimental values[27] for these decay rates are: $\Gamma(\chi_{c0} \rightarrow \gamma\gamma) = 2.9 \pm 0.4$ keV and $\Gamma(\chi_{c2} \rightarrow \gamma\gamma) = 0.534 \pm 0.050$ keV, which can indeed be considered as somewhat suppressed in comparison with $\Gamma(\eta_c \rightarrow \gamma\gamma)$. As usual, some of the theoretical uncertainty goes away if one considers an appropriate ratio, in this case the ratio of the two decay rates in Eq.(28):

$$\frac{\Gamma({}^3P_2 \rightarrow \gamma\gamma)}{\Gamma({}^3P_0 \rightarrow \gamma\gamma)} = \frac{4}{15} \left[1 - \frac{\alpha_s}{3\pi} \left(\pi^2 + \frac{20}{3} \right) \right] = \frac{4}{15} \left(1 - 5.51 \frac{\alpha_s}{\pi} \right). \quad (29)$$

The experimental value of the ratio calculated from the world averages[27] for the decay rates, 0.185 ± 0.025 is indeed smaller than the uncorrected value $4/15 = 0.267$ in Eq.(29), which agrees with the negative value of the QCD correction. However, the latest data with the best precision in a single experiment[?] give $0.235 \pm 0.042 \pm 0.005 \pm 0.030$. In either case it would still be troublesome to draw a more quantitative conclusion due to the large coefficient of the first radiative term.

The same considerations regarding the annihilation to $\gamma\gamma$ can be applied to the recently observed state $Z(3930)$ interpreted as the radial excitation χ'_{c2} . The observation of this resonance[35] is in fact due to its discussed coupling to 2γ . However no quantitative data on the electromagnetic decay width are available so far.

The subject of the two-photon annihilation may also become of relevance for the yet unobserved 1D_2 (2^-) state of charmonium. If the mass of this resonance is below the $D\bar{D}^*$ threshold it should be quite narrow since its unnatural spin-parity forbids decay into $D\bar{D}$ pairs. No mixing due to the tensor forces is possible for this state, therefore its two-photon annihilation would provide an access to the amplitude of direct annihilation from D -wave. A similar amplitude enters the previously discussed annihilation of a 3D_1 state into e^+e^- , where however it gets tangled with the $S - D$ mixing effects. Thus a measurement of the decay ${}^1D_2 \rightarrow \gamma\gamma$ can be helpful in separating the direct annihilation from mixing in the properties of $\psi(3770)$. The rate of the decay is given[4] as

$$\Gamma({}^1D_2 \rightarrow \gamma\gamma) = \frac{2^6 3 e_c^4 \alpha^2}{M^6} |R''_{nD}(0)|^2 \quad (30)$$

in terms of the second derivative at the origin of the same radial function as in Eq.(26).

3.1.4 $J/\psi \rightarrow \gamma\gamma\gamma$

The decays ${}^3S_1 \rightarrow 3\gamma$ have very small rates proportional to α^3 . However a measurement of such decay for the J/ψ resonance does not appear to be unrealistic. The rate of the decay in the potential approach is given by

$$\Gamma(J/\psi \rightarrow 3\gamma) = \frac{16 (\pi^2 - 9) e_c^6 \alpha^3}{3\pi M^2} |R_{1S}(0)|^2 \left(1 - 12.6 \frac{\alpha_s}{\pi} \right), \quad (31)$$

where the lowest-order result is an adaptation of the orthopositronium decay formula[73] and the first QCD correction is also an adaptation[65] of the numerical result[74] for the one-loop QED correction to the orthopositronium decay rate. As usual, model-dependence can be reduced by considering the ratio of this decay rate to Γ_{ee} (Eq.(23)):

$$\frac{\Gamma(J/\psi \rightarrow 3\gamma)}{\Gamma_{ee}(J/\psi)} = \frac{64 (\pi^2 - 9)}{243\pi} \alpha \left(1 - 7.3 \frac{\alpha_s}{\pi} \right) \approx 5.3 \times 10^{-4} \left(1 - 7.3 \frac{\alpha_s}{\pi} \right). \quad (32)$$

The QCD radiative correction in this estimate is large, and it is not clear what numerical value it should be assigned. The uncorrected number puts the three-photon decay rate in the ballpark of 3 eV

corresponding to the branching fraction $\mathcal{B}(J/\psi \rightarrow 3\gamma) \sim 3 \times 10^{-5}$, which can be compared with the current upper limit (at 90% CL): 5.5×10^{-5} .

The three-photon decay, inspite of its small rate, presents a very interesting object for a study of dynamics of charmonium. Indeed, the ratio of the rates in Eq.(32) is sensitive to only the QCD corrections, so that a measurement of this ratio can possibly shed some light on understanding of the behavior of the QCD radiative effects in a situation where the coefficients in the loop expansion are large. Since such behavior of the expansion coefficients is typical for various effects in heavy quarkonium a better understanding of the QCD expansion can be of help in considering those other effects as well. Furthermore, it would be of great interest if one-photon energy spectrum in the 3γ decay could be studied, since the photons provide a very clean ‘‘CT scan’’ of the internal structure of J/ψ . Namely, the photon spectrum at energy ω provides sensitivity to distances $\sim 1/\sqrt{m_c\omega}$, so that dynamics of the charmed quarks can be probed by this spectrum at distances ranging from the short ones, $\sim 1/m_c$, up to the typical charmonium size.

3.2 Strong Annihilation into Light Hadrons

Charmonium decay into light hadrons through the strong interaction is viewed in QCD as a two-stage factorized process. First the $c\bar{c}$ pair annihilates into gluons at ‘short’ distances of order $1/m_c$ and then the gluons fragment into specific hadronic final states. The total inclusive probability of the latter fragmentation is considered to be equal to one, so that the total decay rate is determined by the short-distance annihilation to on-shell gluons. Clearly, this approach based on a perfect ‘gluon - hadron duality’, involves an uncertainty of its own in addition to the previously mentioned uncertainties involved in calculation of charmonium annihilation. Inspite of this reservation, such treatment first considered in Ref. [75], is extremely successful in explaining, at least semi-quantitatively, the pattern of OZI violating narrow widths of the charmonium resonances below the open charm threshold, in particular in understanding the very narrow width of the J/ψ resonance, which has so profoundly awed particle physicists in November 1974.

3.2.1 Three-gluon Annihilation of 3S_1 Charmonium.

The minimal number of gluons into which a 3S_1 state of a heavy quark pair can annihilate is three, since the process through one virtual gluon is forbidden by color and a two-gluon final state is excluded by the negative C parity of the initial state. The decay rate in the lowest order in QCD[75] can be found by decorating the orthopositronium decay formula[73] with the appropriate color factor, while the first QCD radiative correction is known only numerically[76], and being expressed in terms of α_s normalized at the scale m_c in the \overline{MS} scheme reads as

$$\Gamma(n^3S_1 \rightarrow 3g \rightarrow \text{light hadrons}) = \frac{40}{81} \frac{\pi^2 - 9}{\pi} \frac{\alpha_s^3(m_c)}{M^2} |R_{nS}(0)|^2 \left(1 - 3.7 \frac{\alpha_s}{\pi}\right). \quad (33)$$

The ratio of this rate to Γ_{ee} is then sensitive, in this approach, to only the coupling constants:

$$\frac{\Gamma(n^3S_1 \rightarrow 3g \rightarrow \text{light hadrons})}{\Gamma_{ee}(n^3S_1)} = \frac{5}{18} \frac{\pi^2 - 9}{\pi} \frac{\alpha_s^3(m_c)}{\alpha^2} \left(1 + 1.6 \frac{\alpha_s}{\pi}\right). \quad (34)$$

One is naturally tempted to use this formula for an estimate of the QCD coupling α_s . In order to make such estimate one has to evaluate the branching fraction for the direct strong annihilation of J/ψ , which amounts to subtraction from the total sum the electromagnetic annihilation contribution, estimated in Eq.(24), the small contribution of the decay $J/\psi \rightarrow \gamma\eta_c$ with the branching fraction[27] $(1.3 \pm 0.4)\%$, and the contribution of the direct photon emission, originating in the process $J/\psi \rightarrow \gamma gg$, which will be discussed in some detail few lines below. The branching fraction for the latter process is

somewhat uncertain experimentally due to a large background of secondary photons in the low-energy part of the photon spectrum. Namely, the direct photon emission has been observed and reliably measured[77] at $x > 0.6$, where $x = 2\omega_\gamma/M_{J/\psi}$ is the ratio of the photon energy to the maximally allowed by kinematics. The integral over the observed spectrum in this region corresponds to the branching fraction $\mathcal{B}(J/\psi \rightarrow \gamma gg)|_{x>0.6} = (4.1 \pm 0.8)\%$. The experimental shape of the spectrum reasonably suggests that the observed part makes about one half of the total decay rate with another half being hidden under the background at $x < 0.6$. Thus one can rather conservatively estimate the total branching fraction as $\mathcal{B}(J/\psi \rightarrow \gamma gg) \approx (8 \pm 3)\%$. As a result the fraction of the direct strong annihilation of J/ψ can be estimated as $\mathcal{B}(J/\psi \rightarrow 3g) \approx (66 \pm 3)\%$, and the ratio in Eq.(34) as²

$$\frac{\mathcal{B}(J/\psi \rightarrow 3g)}{\mathcal{B}_{ee}(J/\psi)} = 11.1 \pm 0.5 , \quad (35)$$

thus providing one with the estimate $\alpha_s(m_c) \approx 0.19$. It would however be difficult to assign a reliable error bar to this number, given the reservations mentioned previously regarding the assumptions and approximations made in connection with the described calculations of the annihilation rates.

A marginal suitability of the direct hadronic decay rate of charmonium for precision determination of QCD parameters can also be illustrated by comparing the data on the annihilation decays of J/ψ and ψ' . The proportionality of the annihilation rates to $|R_{nS}(0)|^2$ implies a similarity between the decays of J/ψ and ψ' . Namely, the ratios between these two resonances of their similar decay rates should be the all equal to each other. In particular the ratio of the branching fractions for similar decays should all be equal to $\mathcal{B}_{ee}(\psi')/\mathcal{B}(J/\psi) = (12.4 \pm 0.3)\%$ (the so-called “12% rule”). A recent CLEO-c dedicated study[78] of the decays of ψ' ending up in J/ψ in the final state allows to separate the branching fraction for the direct decays of ψ' into light hadrons, which includes the electromagnetic decays of this type and the γgg decays with direct photon: $\mathcal{B}(\psi' \rightarrow \text{lighthadrons}) = (16.9 \pm 2.6)\%$. The same branching fraction for J/ψ is listed in the Tables[27]: $\mathcal{B}(J/\psi \rightarrow \text{lighthadrons}) = (87.7 \pm 0.5)\%$. The ratio of these numbers gives $\mathcal{B}(\psi' \rightarrow \text{lighthadrons})/\mathcal{B}(J/\psi \rightarrow \text{lighthadrons}) = (19.3 \pm 3.0)\%$, which substantially differs from the “12% rule”. One would also arrive at a similar contradiction with the simple picture of charmonium annihilation if first subtracted from $\mathcal{B}(\psi' \rightarrow \text{lighthadrons})$ the small contribution of the electromagnetic processes $\psi' \rightarrow \gamma^* \rightarrow \text{lighthadrons}$ and the (estimated) fraction of $\psi' \rightarrow \gamma gg$, leaving $\mathcal{B}(\psi' \rightarrow 3g) \approx (14.3 \pm 2.9)\%$, which gives the estimate for the ratio of the ggg and e^+e^- annihilation rates of ψ' equal to 19.5 ± 4.0 , i.e. different from the same ratio for J/ψ in Eq.(35).

That there is more dynamics to the hadronic annihilation of the 3S_1 charmonium than the simple factorized approach suggests, is indicated by that the 12% rule is also quite conspicuously broken in exclusive decay modes, the most famous in this respect being the decay into $\rho\pi$ which is strongly suppressed for ψ' , well below the “12% rule”. A number of other exclusive modes with a (seemingly random) deviation from this rule have been measured experimentally[27, 79]. Although deviations from the simple picture are expected on general grounds, the specific mechanisms for such deviations are currently unknown even at the level of inclusive decay rates. We will discuss some considerations on this subject at the end of the current section on charmonium annihilation.

3.2.2 The Decay into γgg

The mixed electromagnetic and strong annihilation into a photon and two gluons, briefly discussed above in connection with the estimate in Eq.(35), is actually quite interesting on its own[80]. The ratio of the total decay rate to that of the decay into e^+e^- in the simple annihilation picture depends only on the QED and QCD coupling constants:

$$\frac{\Gamma(n^3S_1 \rightarrow \gamma gg)}{\Gamma_{ee}(n^3S_1)} = \frac{8}{9} \frac{\pi^2 - 9}{\pi} \frac{\alpha_s^2(m_c)}{\alpha} \left(1 - 1.3 \frac{\alpha_s}{\pi}\right) , \quad (36)$$

²This estimate also agrees with the result of a recent similar evaluation[6] done in a slightly different way.

where the QCD radiative correction is from the result of numerical calculation of Ref. [76]. Using the known value of $\mathcal{B}_{ee}(J/\psi)$ and $\alpha_s(m_c) \approx 0.19$ as estimated from Eq.(35), one can estimate $\mathcal{B}(J/\psi \rightarrow \gamma gg) \approx 6.7\%$, which is in a reasonable agreement with the available data[77].

Besides the total rate of this radiative hadronic decay, of a great interest is the spectrum of the direct photons[4], since this spectrum provides some insight into how the gluon - hadron duality sets in. Indeed the invariant mass squared, q^2 , of the hadronic final state into which the two gluons fragment is related to the photon energy ω as $q^2 = M_{J/\psi}^2(1-x)$, where $x = 2\omega/M_{J/\psi}$. The ‘parton’ spectrum, corresponding to on-shell gluons extends all the way to the kinematical boundary at $x = 1$, corresponding to $q^2 = 0$. In reality, one certainly does not expect a gluon-hadron duality at low q^2 , so that the spectrum of direct photons at high energy should be suppressed by hadronic effects, as is observed in experiment[77]. At larger q^2 , corresponding to lower x , the actual spectrum should approach the parton curve due to the onset of the gluon-hadron duality. It should be noted however that, as previously mentioned, the direct photon spectrum at lower x (in practice at $x \approx 0.5$ for charmonium) goes under an overwhelming background of secondary photons which are present in hadronic final states. Thus in the decays of charmonium it is unlikely that the fragmentation can be studied at q^2 beyond 4 or 5 GeV², and a much better testing ground for such study is provided by the decays of the bottomonium Υ resonances[81].

3.2.3 Two-gluon Annihilation of C -even States

The C -even states of quarkonium with $J \neq 1$ can annihilate into two gluons, much in the same way as they decay into two photons. In fact in the lowest order the gg and $\gamma\gamma$ decay rates are simply related[64, 4] as $\Gamma_{gg}/\Gamma_{\gamma\gamma} = 2\alpha_s^2/(9e_c^4\alpha^2) = (9/8)(\alpha_s/\alpha)^2$. The coefficients of the first QCD correction are, naturally, different for individual states[66, 71]:

$$\frac{\Gamma(^1S_0 \rightarrow gg)}{\Gamma(^1S_0 \rightarrow \gamma\gamma)} = \frac{9}{8} \left[\frac{\alpha_s^2(m_c)}{\alpha} \right]^2 \left(1 + 8.2 \frac{\alpha_s}{\pi} \right), \quad (37)$$

$$\frac{\Gamma(^3P_0 \rightarrow gg)}{\Gamma(^3P_0 \rightarrow \gamma\gamma)} = \frac{9}{8} \left[\frac{\alpha_s^2(m_c)}{\alpha} \right]^2 \left(1 + 9.3 \frac{\alpha_s}{\pi} \right), \quad \frac{\Gamma(^3P_2 \rightarrow gg)}{\Gamma(^3P_2 \rightarrow \gamma\gamma)} = \frac{9}{8} \left[\frac{\alpha_s^2(m_c)}{\alpha} \right]^2 \left(1 + 3.1 \frac{\alpha_s}{\pi} \right). \quad (38)$$

The correction for the decay rate of the 1D_2 state is presently unknown. One can see that the coefficients of the first QCD correction for 1S_0 and 3P_0 states are unusually large, which makes difficult a reliable quantitative comparison with the data. The uncorrected formulas with $\alpha_s \approx 0.2$ would give for the ratio of the width the value of about 850, which is not even close to the observed values[27]: $\Gamma_{gg}(\eta_c)/\Gamma_{\gamma\gamma}(\eta_c) = (3.57 \pm 1.15) \times 10^3$, $\Gamma_{gg}(\chi_{c0})/\Gamma_{\gamma\gamma}(\chi_{c0}) = (3.62 \pm 0.43) \times 10^3$, and $\Gamma_{gg}(\chi_{c2})/\Gamma_{\gamma\gamma}(\chi_{c2}) = (3.09 \pm 0.23) \times 10^3$. The experimental data can be somewhat understood using a significantly larger QCD coupling $\alpha_s(m_c) \approx 0.3$ than is inferred from the three-gluon decays of J/ψ . Such larger value of $\alpha_s(m_c)$ better complies with the determination[82] of the QCD coupling from τ decays and e^+e^- annihilation data, but then it entirely misses the observed rate of the three-gluon annihilation. It is not clear at present how to avoid this crunch. One could possibly argue for the η_c and χ_{c0} resonances that their hadronic decay rates are enhanced by the instanton-type quarkonium glue mixing in the 0^{-+} and 0^{++} channels[59]. However such argument would still leave the observed $2g/2\gamma$ ratio for the χ_{c2} unexplained.

3.2.4 Strong Annihilation of 1P_1 and 3P_1 States

The so far discussed amplitudes of the decays into on-shell gluons are not sensitive to the infrared behavior of quarks and gluons, at least at the considered here level of perturbative calculation at the leading and the one-loop level. In other words, these amplitudes are all expressed in terms of the wave function of the quark pair at the typical annihilation distances of order $1/m_c$. In the two-gluon decays this behavior is guaranteed by the fixed and large energy of each of the gluons in the c.m. system,

while for the three-gluon decays of the 3S_1 states the kinematical region, where one of the gluons is soft, is not enhanced due to considering the heavy quarks being essentially static and thus not radiating soft gluons. Clearly, such “good” infrared behavior generally becomes invalid in higher orders of the perturbative expansion, as well as beyond the leading nonrelativistic approximation. For the 1P_1 and 3P_1 states the infrared behavior of the amplitudes describing their annihilation shows up already in the leading approximation[83]. The reason for the infrared sensitivity of these amplitudes is that a P wave necessarily involves a motion of the quarks, so that a P -wave state can go into an S -wave by an E1 emission of a soft gluon. The spin of the quarks is not involved in the emission. Therefore the colorless 1P_1 state goes into a color-octet 1S_0 state of the quark pair, while the 3P_1 state goes into a color-octet 3S_1 . The intermediate 1S_0 state decays into two gluons, while the colored 3S_1 can decay either into a light quark-antiquark pair, $q\bar{q}$ through a virtual gluon, or, generally, into a gluon pair. It turns out however[83, 4] that for a static colored 3S_1 pair of heavy quarks the amplitude of annihilation into two gluons is identically zero in the leading order, thus the resulting decay processes are $^1P_1 \rightarrow ggg$ and $^3P_1 \rightarrow gq\bar{q}$. In calculating the total probability one has to integrate over the energy ω of the soft gluon, which integration, as is standard for a soft gauge quantum emission, is infrared-divergent as $\int d\omega/\omega$. With a logarithmic accuracy the upper limit in this integral is set by the quark mass, and the lower limit is a typical QCD mass scale.

The leading-logarithm expressions for the hadronic widths of the 1P_1 and 3P_1 states are given as[83, 4]

$$\Gamma(^1P_1 \rightarrow 3g \rightarrow \text{hadrons}) = \frac{20}{9} \frac{\alpha_s^3}{m_c^4} |R'_P(0)|^2 \ln \frac{m_c}{\Lambda_{QCD}} , \quad (39)$$

and

$$\Gamma(^3P_1 \rightarrow \text{hadrons}) = \sum_{q=u,d,s} \Gamma(^3P_1 \rightarrow g q\bar{q} \rightarrow \text{hadrons}) = \frac{8}{3\pi} \frac{\alpha_s^3}{m_c^4} |R'_P(0)|^2 \ln \frac{m_c}{\Lambda_{QCD}} . \quad (40)$$

One can see that the numerical coefficients in these formulas correspond to the hadronic width of the 1P_1 being larger than that of the 3P_1 by the factor $5\pi/6 \approx 2.6$. The experimental value of the hadronic width of the 1P_1 charmonium resonance, χ_{c1} , is approximately 0.6 MeV, so that based on these formulas one would expect the hadronic width of the 1P_1 resonance, h_c , to be about 1.5 MeV. The latter value does not contradict the CLEO-c data[30], but it might be in a contradiction with the Fermilab E835 results[29] claiming the width of the observed h_c resonance to be $\Gamma \leq 1$ MeV. Apriori one would expect the applicability of these logarithmic estimates of the annihilation rates to be marginal at best.

3.3 Non-perturbative Effects in Hadronic Annihilation

The logarithmic formulas become formally applicable in the limit of very heavy quarks, where the resulting logarithm can be considered as a large parameter, and one can develop a formalism[84] of a logarithmic theory of the processes with soft-gluon transitions between color-octet and color-singlet heavy quarkonium states, or consider purely phenomenologically the mixing between pure colorless $c\bar{c}$ states and states where a color-octet quark pair is accompanied by a soft gluon field. An extensive overview of this approach, called in the literature Non-Relativistic QCD (NRQCD), and a list of references can be found in the review [5].

The effects of soft gluons can be addressed not only in the decays of the 3P_1 and 1P_1 states, where such effects are dominant, but also for ‘well behaved’ processes, where these effects give rise to small or moderate corrections. In particular the corrections arising from nonperturbative soft gluon field in the annihilation of the 3S_1 states may be responsible for the previously discussed deviations from predictions based on the simple perturbative picture, such as the “12% rule”. A consistent analysis of the leading nonperturbative corrections to the annihilation rates is however possible only in the limit of very heavy quarkonium, for which such corrections are expressed[85] in terms of the gluon vacuum condensate.

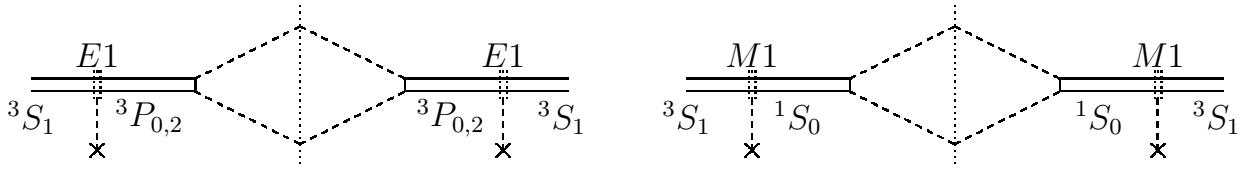


Figure 3: The mechanism describing the leading nonperturbative correction to the annihilation rate of a heavy 3S_1 quarkonium shown as a unitary cut across two hard gluons. The soft gluons (ending in a “ \times ”) are described by the field in the vacuum condensate $\langle 0|G^2|0\rangle$.

The leading nonperturbative corrections to the three-gluon annihilation rate of a heavy 3S_1 quarkonium are described by the mechanism shown in Fig.3. The soft gluon field converts the color-singlet 3S_1 state into a color-octet $^3P_{0,2}$ state through the $E1$ chromoelectric interaction, or into a color-octet 1S_0 state due to the chromomagnetic $M1$ interaction. The final state in either of these transitions then decays into two hard gluons. The contribution of this mechanism can be calculated as the imaginary part of the graphs shown in Fig.3 given by the unitarity cut across the hard gluons, while the averaging of the quadratic expression in the soft gluon field makes the effect proportional to the gluon condensate $\langle 0|G^2|0\rangle$.

It is quite essential that both the contributions of the 3P_J and 1S_0 are comparable even for a non-relativistic quarkonium. Indeed, the chromomagnetic $M1$ transition to the 1S_0 state is suppressed by the inverse of the heavy quark mass, $1/m_Q$, in the amplitude, while the amplitude of annihilation of the $^3P_{0,2}$ states contains an extra factor of $1/m_Q$ in comparison with that for the 1S_0 . Furthermore, the contribution of the 3P_J intermediate states is proportional to the vacuum condensate of the chromoelectric field, $\langle 0|\vec{E}^2|0\rangle$, while that of the 1S_0 intermediate state is given by the chromomagnetic condensate, $\langle 0|\vec{B}^2|0\rangle$. The Lorentz invariance of the vacuum however requires that these two condensates have opposite sign[12]:

$$\langle 0|\vec{B}^2|0\rangle = -\langle 0|\vec{E}^2|0\rangle = \frac{1}{4} \langle 0|G^2|0\rangle . \quad (41)$$

The important feature of the discussed mechanism is that its dependence on the wave function of the 3S_1 state is not reduced to $\psi(0)$, but rather is determined by the corresponding overlap integrals. Therefore such mechanism generally breaks the similarity between the decays of different 3S_1 states. The same behavior however implies that the overlap integrals can be reliably evaluated only for an asymptotically heavy quarkonium, whose dynamics is dominated by the Coulomb-like potential due to the gluon exchange between the quark and the antiquark. In this limit one finds for the relative correction to the decay rate of an n^3S_1 state the expression[85]

$$\frac{\delta\Gamma}{\Gamma}(n^3S_1) = -\frac{\pi\langle 0|G^2|0\rangle}{2^{11} 9 (\pi^2 - 9) \alpha_s(m_Q) k_n^4} \frac{a_n}{n^4} , \quad (42)$$

where a_n are numerical coefficients given by

$$a_n = \frac{2^4 7 n^4}{9} \left[\frac{64 (n^2 - 64)}{(81 n^2 - 256) (81 n^2 - 64)} \right]^2 - 2^{14} 9 \frac{n^2}{(81 n^2 - 64)^2} , \quad (43)$$

and k^n is the ‘Bohr momentum’ for the n -th state determined from the relation

$$k_n = \frac{4}{3 n} m_Q \alpha_s(k_n) . \quad (44)$$

It should be mentioned that the normalization gluon field $G_{\mu\nu}$ used here corresponds to the gluon term in the QCD Lagrangian of the form $L = -(1/4g^2)G^2$, so that numerically the gluon condensate is $\langle 0|G^2|0 \rangle \approx 0.7 \text{ GeV}^4$.

As previously discussed the chromoelectric and the chromomagnetic contributions have opposite sign, so that the coefficients a_n in Eq.(43), have the form of a difference: $a_n = a_n(E) - a_n(M)$. Furthermore, the coefficients $a_n(E)$ at $n \geq 3$ and $a_n(M)$ at $n \geq 2$ are very small. This is a consequence of a relatively weak Coulomb-like interaction in the octet state. If the latter interaction were neglected altogether (the $N_c \rightarrow \infty$ limit) those coefficients would also vanish, and the only remaining nonzero would be $a_1(E) = a_2(E) = 28$ and $a_1(M) = 27$. The real world values given by Eq.(43) are $a_1 = 22.858 - 18.897 = 3.961$ and $a_2 = 9.393 - 0.323 = 9.070$. These values illustrate that for the ground 3S_1 state the discussed correction is very sensitive to the details of the quarkonium dynamics, so that any extrapolation of the asymptotic formulas down to the realistic bottomonium and charmonium necessarily is quite uncertain.

If such extrapolation is done by using $k_1 \approx 1 \text{ GeV}$ for the Υ resonance and $k_1 \approx 0.5 \text{ GeV}$ for J/ψ , one would estimate the relative effect in the hadronic annihilation of Υ as about 0.5 %, and about 5 % for the J/ψ resonance. If however the cancellation between the chromoelectric and chromomagnetic terms does not occur for these realistic quarkonia, the effect can be few times larger.

A similar mechanism, with one of the hard gluons replaced by photon, describes the nonperturbative correction to the rate of the decay ${}^3S_1 \rightarrow \gamma + 2g$. The relative correction in this case is three times smaller than in Eq.(42).

4 Radiative Transitions

The treatment of charmonium as a nonrelativistic system suggests that one can apply the standard multipole expansion in electrodynamics to calculation of the transitions between charmonium levels with emission of a photon. The leading terms in this expansion are the $E1$ and $M1$ terms, which can be described for the discussed here transitions by the corresponding terms in the Hamiltonian:

$$H_{E1} = -e_c e (\vec{r} \cdot \vec{E}) \quad \text{and} \quad H_{M1} = -\mu_c (\vec{\Delta} \cdot \vec{B}) , \quad (45)$$

where $e_c = 2/3$ is the charge of the charmed quark, $\mu_c = e_c e / (2m_c)$ is its magnetic moment, \vec{E} and \vec{B} are standing for the electric and the magnetic field, and $\vec{\Delta}$ being a spin operator defined as $\vec{\Delta} = \vec{\sigma}_1 - \vec{\sigma}_2$ with $\vec{\sigma}_1$ and $\vec{\sigma}_2$ acting respectively on the quark and the antiquark. The electric dipole term is responsible for the transitions between the S and P states with the same spin S of the quark pair, while the $M1$ term describes the transitions between $S = 1$ and $S = 0$ states with the same orbital momentum L .

4.1 $E1$ Transitions

4.1.1 $\psi' \rightarrow \gamma \chi_{cJ}$

The rate for the transitions from a 3S_1 state to 3P_J induced by the $E1$ term in Eq.(45) is given by

$$\Gamma({}^3S_1 \rightarrow \gamma {}^3P_J) = (2J+1) \frac{4}{27} e_c^2 \alpha \omega_\gamma^3 |I_{PS}|^2 , \quad (46)$$

where ω_γ is the energy of the emitted photon, and I_{PS} is the radial overlap integral:

$$I_{PS} = \langle P | r | S \rangle = \int_0^\infty r^3 R_P(r) R_S(r) dr , \quad (47)$$

with $R_{S,P}(r)$ being the normalized radial wave functions for the corresponding states.

Clearly, the overlap integral has dimension of length, and its particular value is somewhat model dependent.(With certain restrictions following from general Quantum Mechanical considerations[86, 4].) One can however assess the overall validity of the nonrelativistic formula (46) for charmonium by comparing with each other the experimentally known rates of the transitions from ψ' to the χ_{cJ} levels with different J . This can be done in terms of the value of the overlap integral, $|I_{PS}|$, extracted from the rate of each of the transition. Proceeding in this way one finds

$$|\langle 1P|r|2S\rangle| \approx \begin{cases} 0.37 \text{ fm from } \psi' \rightarrow \chi_{c0} \gamma \\ 0.39 \text{ fm from } \psi' \rightarrow \chi_{c1} \gamma \\ 0.45 \text{ fm from } \psi' \rightarrow \chi_{c2} \gamma , \end{cases} \quad (48)$$

so that the extracted values of the overlap integral agree with each other within the expected accuracy of the expression (46), and the absolute value of the radial integral also agrees well with a general understanding of the typical size of charmonium.

4.1.2 $\chi_{cJ} \rightarrow \gamma J/\psi$

The transitions from 3P_J levels to a 3S_1 state are described by the expression for the rate

$$\Gamma({}^3P_J \rightarrow \gamma {}^3S_1) = \frac{4}{9} e_c^2 \alpha \omega_\gamma^3 |I_{SP}|^2 . \quad (49)$$

Performing a similar to previous extraction of the overlap integral from the experimental data on the rates of the $\chi_{cJ} \rightarrow J/\psi$ decays, one finds the overlap integral for the $1P \rightarrow 1S$ transitions as

$$|\langle 1S|r|1P\rangle| = \begin{cases} 0.36 \text{ fm from } \chi_{c0} \rightarrow J/\psi \gamma \\ 0.38 \text{ fm from } \chi_{c1} \rightarrow J/\psi \gamma \\ 0.37 \text{ fm from } \chi_{c2} \rightarrow J/\psi \gamma . \end{cases} \quad (50)$$

Thus one can conclude that the observed $E1$ transitions in charmonium present us with no unusual behavior.

4.1.3 $h_c \rightarrow \gamma \eta_c$

The decay $h_c \rightarrow \gamma \eta_c$ is crucial for identifying the charmonium 1P_1 resonance in the experiments[29, 30]. A straightforward application of the $E1$ transition formula results in the expression for the rate of this decay

$$\Gamma({}^1P_1 \rightarrow \gamma {}^1S_0) = \frac{4}{9} e_c^2 \alpha \omega_\gamma^3 |I_{SP}|^2 , \quad (51)$$

where the overlap integral is the same as is just estimated from the $\chi_{cJ} \rightarrow \gamma J/\psi$ decays, up to the relativistic corrections $O(v^2/c^2)$. Thus using the extracted value of $|I_{SP}|$ one can readily predict the width of the decay in absolute terms:

$$\Gamma(h_c \rightarrow \gamma \eta_c) \approx 0.65 \text{ MeV} . \quad (52)$$

As discussed previously, the hadronic width of the h_c is quite uncertain. The logarithmic formulas and the data on the hadronic width of χ_{c1} indicate that hadronic decay rate of h_c can be as large as 1.5 MeV, in which case the radiative decay branching ratio should be about 30%. It is believed that this branching ratio, with all the uncertainties involved, should be in the range between 30% and 50%.

4.1.4 Relativistic Effects

In the next order of the relativistic expansion the leading amplitudes of the considered (dominantly) $E1$ transitions, receive $O(v^2/c^2)$ corrections from the magnetic quadrupole $M2$ and the electric octupole $E3$ terms[87, 88]. These corrections are expected to be suppressed relative to the leading term by a factor $v^2/c^2 \approx 0.2$. In the total decay rate these terms do not interfere with the leading one, so that their contribution is only of order few percent and hardly can be measured. However these amplitudes generally interfere with the $E1$ term in the angular distribution. In particular for the $1P \rightarrow 1S$ transitions the $M2$ and $E3$ contributions can be studied by measurements of the angular distributions and helicity amplitudes in the processes $p\bar{p} \rightarrow \chi_{c1,2} \rightarrow \gamma J/\psi$. The most precise available measurements were performed by the E835 experiment[89]. The current averages[27] for the relative values of the amplitudes are: $M2/|E1| = -0.13 \pm 0.05$ and $E3/|E1| = 0.011^{+0.041}_{-0.033}$ from the $\chi_{c2} \rightarrow \gamma J/\psi$ decays, and $M2/|E1| = -0.002^{+0.008}_{-0.017}$ from $\chi_{c1} \rightarrow \gamma J/\psi$.

Thus the data on the radiative decay of the χ_{c2} generally support the expectations: the suppression of the $M2$ amplitude is indeed of order v^2/c^2 , while the $E3$ amplitude was predicted to be small[88]. On the other hand the result of the extraction of $M2$ from the χ_{c1} decay is neither compatible with that for the χ_{c2} , nor does it comply with theoretical expectations. Clearly, this situation strongly suggests that further studies of the relativistic terms are needed.

4.2 $M1$ Transitions and the Puzzle of η_c

The radiative transitions induced by the $M1$ term (Eq.(45)) appear to be simpler than the $E1$ ones, since the $M1$ interaction does not contain any coordinate dependence, while the spin matrix elements are trivial inasmuch as the spin and coordinate degrees of freedom are factorized. Phenomenologically the most interesting are the $M1$ transitions $J/\psi \rightarrow \gamma \eta_c$ and $\psi' \rightarrow \gamma \eta_c$, whose rates are given by

$$\Gamma(n^3S_1 \rightarrow \gamma m^1S_0) = \frac{4}{3} e_c^2 \alpha \frac{\omega_\gamma^3}{m_c^2} |I_{mn}|^2, \quad (53)$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final charmonium states. In the leading nonrelativistic order, where there is no effect of the spin-spin interaction on the coordinate wave function, the overlap integrals are determined by the orthonormality condition for the coordinate wave functions: $I_{mn} = \delta_{mn}$. Using for an estimate $m_c = 1.4$ GeV one thus calculates the rate of the transition $J/\psi \rightarrow \gamma \eta_c$ as $\Gamma(J/\psi \rightarrow \gamma \eta_c) \approx 3.3$ keV, while the overlap matrix element vanishes for the $2S \rightarrow 1S$ transition $\psi' \rightarrow \gamma \eta_c$. The latter decay becomes possible due to the relativistic effects, but at present any estimate of the actual rate would be quite unreliable.

In reality both decays have very similar rates: $\Gamma(J/\psi \rightarrow \gamma \eta_c) \approx 1.2$ keV and $\Gamma(\psi' \rightarrow \gamma \eta_c) \approx 0.9$ keV. Although for the latter transition the small value of the rate is not surprising and can be readily attributed to the suppression of the relativistic effects, the failure of the seemingly robust theoretical expectation for the former transition is quite paradoxical.

One can certainly attempt to explain the discrepancy of almost a factor of 3 between the experimental value and the straightforward theoretical estimate for the rate of the transition from the J/ψ in terms of a larger charmed quark mass, modified magnetic moment of the quark, and enhanced relativistic effects, or a combination of these factors. However such a ‘post diction’ would inevitably have to be quite contrived. Furthermore, Shifman has presented a phenomenological argument[90], which eliminates the uncertainty related to the charmed quark mass and magnetic moment, and still leaves a considerable gap between the theoretical and the experimental values of the rate. The argument is based on applying the standard dispersion relation to the amplitude of the decay $\eta_c \rightarrow \gamma\gamma$. Namely, this amplitude is

described by just one form factor F :

$$A(\eta_c \rightarrow \gamma\gamma) = F \varepsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^{(1)} F_{\lambda\sigma}^{(2)} \quad (54)$$

with $F_{\mu\nu}^{(1,2)}$ being the field tensors for the two photons. The form factor is a function of the squares of the three 4-momenta, involved in the process: $F(m_{\eta_c}^2, k_1^2, k_2^2)$, and the width is determined by its value for on-shell photons: $\Gamma(\eta_c \rightarrow \gamma\gamma) = |F(m_{\eta_c}^2, 0, 0)|^2 m_{\eta_c}^3 / 4\pi$. One can further use the dispersion relation for the form factor in one of the photon momenta:

$$F(m_{\eta_c}^2, 0, k^2) = \frac{1}{\pi} \int \frac{ds}{s - k^2} \text{Im}F(m_{\eta_c}^2, 0, s) , \quad (55)$$

and express the imaginary part in the integrand in terms of contribution of real intermediate states. These states with quantum numbers $J^{PC} = 1^{--}$ include the J/ψ resonance, higher charmonium resonances: ψ' , $\psi(3770)$ and so on, and the continuum of the states with charmed $D(D^*)$ mesons.

The contribution of the J/ψ is expressed in terms of the product of the amplitudes $A(\eta_c \rightarrow \gamma J/\psi) A(J/\psi \rightarrow \gamma^*(s))$, where the coupling of J/ψ to a virtual photon is obviously related to its decay width into e^+e^- . The contribution of the higher charmonium resonances is vanishing, as discussed, in the leading nonrelativistic approximation and is indeed very small phenomenologically. Namely, the effective overlap integral I_{21} , that would fit the observed rate of the decay $\psi' \rightarrow \gamma \eta_c$, is numerically only about 0.04. Similarly the contribution of the continuum to the integral in Eq.(55) can also be argued[90] to be quite small compared to the contribution of the J/ψ . Therefore the dispersion relation (55) should be saturated to a good accuracy by the J/ψ pole thus relating the amplitudes of the decays $\eta_c \rightarrow \gamma\gamma$, $J/\psi \rightarrow e^+e^-$ and $J/\psi \rightarrow \gamma \eta_c$ and the masses of η_c and J/ψ , i.e. without the need to invoke the mass of the charmed quark. The resulting relation between the rates reads as

$$\Gamma(J/\psi \rightarrow \gamma \eta_c) = \frac{2}{9} \frac{\Gamma(\eta_c \rightarrow \gamma\gamma)}{\Gamma(J/\psi \rightarrow e^+e^-)} \alpha \frac{m_{J/\psi}^4}{m_{\eta_c}^3} \left(1 - \frac{m_{\eta_c}^2}{m_{J/\psi}^2}\right)^3 [1 + O(\alpha_s)] . \quad (56)$$

Using the experimental data in the r.h.s. of this relation, one finds the central value of the discussed rate (modulo the QCD corrections) as $\Gamma(J/\psi \rightarrow \gamma \eta_c) = 2.9 \text{ keV}$, which is pretty close to the previous simple nonrelativistic estimate, and is still very far away from the experimental value of this rate.

It may well be that the puzzles of the η_c : its large total width, small branching fraction of decay into $\gamma\gamma$ and the suppression of the decay $J/\psi \rightarrow \gamma \eta_c$ are all tied together and possibly can be resolved by a strong mixing in the 0^{-+} channel[59], so that the η_c has a sizable admixture of light quark and gluon states. Qualitatively such admixture would enhance the hadronic decay rate and suppress the radiative transition from a pure charmonium state J/ψ . In terms of the Shifman's argument, there would arise an additional contribution to the imaginary part of the form factor F from 1^{--} states lighter than J/ψ , which would generally violate the relation (56). However a quantitative description of these phenomena cannot be offered at present.

5 Hadronic Transitions Between Charmonium Resonances

Similarly to the radiative transitions between heavy quarkonium levels, caused by the interaction of quarks with photons, the hadronic transitions arise through the interaction of the heavy quarks with gluons and the gluons materializing as light mesons, the pions and η . Also in complete analogy with the radiative transitions, the interaction of a nonrelativistic quarkonium with the gluon field in hadronic transitions can be described within the multipole expansion in QCD[91]. The leading terms, that will be important in our further discussion of the realistic processes are the chromoelectric and the

chromomagnetic dipoles, $E1$ and $M1$ and the chromomagnetic quadrupole $M2$. The corresponding terms in the Hamiltonian can be written as

$$H_{E1} = -\frac{1}{2}\xi^a \vec{r} \cdot \vec{E}^a(0), \quad H_{M1} = -\frac{1}{2M}\xi^a (\vec{\Delta} \cdot \vec{B}^a), \quad \text{and} \quad H_{M2} = -(4m_Q)^{-1}\xi^a S_j r_i (D_i B_j(0))^a, \quad (57)$$

where $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark (e.g. $t_1^a = \lambda^a/2$ with λ^a being the Gell-Mann matrices), \vec{r} is the vector for relative position of the quark and the antiquark, $\vec{S} = (\vec{\sigma}_Q + \vec{\sigma}_{\bar{Q}})/2$ is the operator of the total spin of the quark-antiquark pair, and \vec{D} is the QCD covariant derivative. Finally \vec{E} and \vec{B} are the chromoelectric and chromomagnetic components of the gluon field strength tensor.

An important difference of the hadronic transitions from the radiative ones is that the physical amplitudes arise in at least the second order in the interaction with the gluon field due to the color requirements. Considering quarkonium as a compact object interacting with soft gluonic fields, one can further approximate the quarkonium transition in the second order in the interaction Hamiltonian in terms of a local colorless gluonic operator, which operator produces the light mesons in the actual hadronic transition. Thus the full transition amplitude factorizes into the heavy quarkonium part determined by the terms in the multipole expansion (Eq.(57)) and the production amplitudes of light mesons by the gluonic operators. The heavy quarkonium part is to a great extent model dependent, and here we discuss this part only by relying on the general dynamical properties, while the production of soft light mesons by local gluonic operators can be described using chiral algebra and certain low-energy theorems in QCD. In what follows we first concentrate on the latter description, and then apply the results to specific hadronic transitions.

5.1 Production of Light Mesons by Gluonic Operators

The structures arising in the second order in the interaction terms in Eq.(57) are quadratic in the gluon field strength tensor. Therefore the interesting for the present consideration amplitudes are those for the production of one or two light pseudoscalar mesons by the operator of the general form $G_{\mu\nu}^a G_{\lambda\sigma}^a$. As will be described, in the low-energy limit the two-meson production by this operator is determined[92, 93], up to a small constant, from the chiral algebra and the QCD anomaly in the trace of the energy-momentum tensor[94, 95, 96, 97], while the one-meson production amplitude is fully fixed by the anomaly in the light quark axial current[98, 99]. Furthermore, the matrix element for one-meson production by the operator containing one extra covariant derivative, $G_{\mu\nu} D G_{\lambda\sigma}$ is entirely determined[92, 100] (in the same low-energy limit) by the fixed matrix element without the derivative, and is therefore relatively well understood. The knowledge of the production amplitudes for the light mesons in fact results in relations between the observed transitions which appear to hold quite well when compared with the data.

5.1.1 Two-pion Production Amplitude

Let us consider first the two-pion production amplitude $\langle \pi^+(p_1)\pi^-(p_2)|G_{\mu\nu}^a G_{\lambda\sigma}^a|0\rangle$ ³. In the leading chiral limit the momenta p_1 and p_2 of the pions as well as the pion mass m_π are to be considered as small parameters, and the expression for the amplitude, quadratic in these parameters, can be written in the following general form

$$-\langle \pi^+(p_1)\pi^-(p_2)|G_{\mu\nu}^a G_{\lambda\sigma}^a|0\rangle = [X(p_1 \cdot p_2) + Y(p_1^2 + p_2^2 - m_\pi^2)] (g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}) + Z t_{\mu\nu\lambda\sigma}, \quad (58)$$

³The charged pions are considered for definiteness. The amplitude for the neutral pion pair production is trivially related by the isospin symmetry.

where the structure

$$\begin{aligned} t_{\mu\nu\lambda\sigma} &= (p_{1\mu}p_{2\lambda} + p_{1\lambda}p_{2\mu}) g_{\nu\sigma} + (p_{1\nu}p_{2\sigma} + p_{1\sigma}p_{2\nu}) g_{\mu\lambda} \\ &- (p_{1\mu}p_{2\sigma} + p_{1\sigma}p_{2\mu}) g_{\nu\lambda} - (p_{1\nu}p_{2\lambda} + p_{1\lambda}p_{2\nu}) g_{\mu\sigma} - (p_1 \cdot p_2) (g_{\mu\lambda}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\lambda}) \end{aligned} \quad (59)$$

has zero overall trace: $t_{\mu\nu}^{\mu\nu} = 0$, and X , Y , and Z are yet undetermined coefficients. The form of the amplitude in Eq.(58) is uniquely determined by the symmetry (with respect to the indices) of the operator $G_{\mu\nu}G_{\lambda\sigma}$ and by the Adler zero condition, which requires that the amplitude goes to zero when either one of the two pion momenta is set to zero and the other one is on the mass shell. One can also notice that the proper index symmetry and the Adler zero condition also automatically ensure that the amplitude is C even, i.e. symmetric under the permutation of the pion momenta: $p_1 \leftrightarrow p_2$.

The coefficients X and Y in Eq.(58) are in fact determined [92] by the anomaly in the trace of the energy-momentum tensor $\theta_{\mu\nu}$ in QCD. Indeed, in the low-energy limit in QCD, i.e. in QCD with three light quarks, one finds

$$\theta_\mu^\mu = -\frac{b}{32\pi^2} G_{\mu\nu}^a G^{a\mu\nu} + \sum_{q=u,d,s} m_q(\bar{q}q) , \quad (60)$$

where $b = 9$ is the first coefficient in the beta function for QCD with three quark flavors. The first term in Eq.(60) represents the conformal anomaly, while the quark mass term arises due to the explicit breaking of the scale invariance by the quark masses. On the other hand, the matrix element of the energy-momentum tensor $\theta_{\mu\nu}$ over the pions: $\theta_{\mu\nu}(p_1, p_2) \equiv \langle \pi^+(p_1)\pi^-(p_2) | \theta_{\mu\nu} | 0 \rangle$ is fully determined [92, 93, 101] in the quadratic in p_1, p_2 and m_π order by the conditions of symmetry in μ and ν , conservation on the mass shell ($(p_1 + p_2)^\mu \theta_{\mu\nu}(p_1, p_2) = 0$ at $p_1^2 = p_2^2 = m_\pi^2$), normalization ($\theta_{\mu\nu}(p, -p) = 2 p_\mu p_\nu$ at $p^2 = m_\pi^2$), and the Adler zero condition ($\theta_{\mu\nu}(p, 0) |_{p^2=m_\pi^2} = 0$):

$$\theta_{\mu\nu}(p_1, p_2) = [(p_1 \cdot p_2) + p_1^2 + p_2^2 - m_\pi^2] g_{\mu\nu} - p_{1\mu}p_{2\nu} - p_{1\nu}p_{2\mu} . \quad (61)$$

The equations (58) and (61) allow for the pion momenta to be off-shell in order to demonstrate the Adler zero. However in what follows only the amplitudes with pions on the mass shell will be considered, so that it will be henceforth implied that $p_1^2 = p_2^2 = m_\pi^2$. In particular one finds for the trace of the expression in Eq.(61)

$$\theta_\mu^\mu(p_1, p_2) = 2(p_1 \cdot p_2) + 4m_\pi^2 . \quad (62)$$

Furthermore, the quark mass term in Eq.(60), corresponding to the explicit breaking of the chiral symmetry in QCD corresponds to the same symmetry breaking by the pion mass term in the pion theory. Thus one finds to the quadratic order in m_π^2 :

$$\langle \pi^+ \pi^- | \sum_{q=u,d} m_q(\bar{q}q) | 0 \rangle = m_\pi^2 , \quad (63)$$

while the term with the strange quark makes no contribution to the discussed amplitude.

Combining the formula in Eq.(60) for θ_μ^μ with the expressions (62) and (63) one finds the matrix element of the gluonic operator over the pions in the form⁴

$$-\langle \pi^+(p_1)\pi^-(p_2) | G_{\mu\nu}^a G^{a\mu\nu} | 0 \rangle = \frac{32\pi^2}{b} [2(p_1 \cdot p_2) + 3m_\pi^2] \quad (64)$$

which thus allows to determine the coefficients X and Y in Eq.(58): $X = 16\pi^2/(3b)$ and $Y = 3X/2 = 8\pi^2/b$.

⁴It can be mentioned that this relation, taking into account the pion mass, was used in Ref. [102], and was also derived in a particular chiral model in Refs. [103, 104].

The coefficient Z of the traceless part in Eq.(58) cannot be found from the trace relation (60). Novikov and Shifman [93] estimated this coefficient by relating this part to the matrix element of the traceless (twist-two) energy-momentum tensor of the gluons only: $\theta_{\mu\nu}^G = 4\pi\alpha_s (-G_{\mu\lambda}^a G_{\nu}^{a\lambda} + \frac{1}{4} g_{\mu\nu} G_{\lambda\sigma}^a G^{a\lambda\sigma})$,

$$Z t_{\mu\lambda\nu}^{\lambda} = 4\pi\alpha_s \langle \pi^+(p_1)\pi^-(p_2) | \theta_{\mu\nu}^G | 0 \rangle . \quad (65)$$

They then assume that the matrix element of the twist-two operator is proportional to the traceless part of the phenomenological energy momentum tensor of the pions,

$$\langle \pi^+(p_1)\pi^-(p_2) | \theta_{\mu\nu}^G | 0 \rangle = \rho_G \left[\frac{1}{2} (p_1 \cdot p_2) g_{\mu\nu} - p_{1\mu} p_{2\nu} - p_{1\nu} p_{2\mu} \right] \quad (66)$$

with the proportionality coefficient interpreted, similarly to the deep inelastic scattering, as “the fraction of the pion momentum carried by gluons”. They further introduce a related parameter $\kappa = b\alpha_s\rho_G/(6\pi)$ and conjecture that numerically $\kappa \approx 0.15 - 0.20$. However, for the purpose of considering the two-pion transitions it is not necessary to pursue the interpretation of κ as being related to the gluon structure function of pion, but rather it can be treated as a phenomenological parameter which can be determined from the data.

Summarizing the results so far in this section one can write the expression for the general matrix element (58) for on-shell pions as

$$-\langle \pi^+(p_1)\pi^-(p_2) | F_{\mu\nu}^a F_{\lambda\sigma}^a | 0 \rangle = \frac{8\pi^2}{3b} \left[(q^2 + m_\pi^2) (g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda}) - \frac{9}{2} \kappa t_{\mu\nu\lambda\sigma} \right] , \quad (67)$$

where $q = p_1 + p_2$ is the total four-momentum of the dipion.

Few remarks are due regarding effects of higher order in α_s . The trace term in Eq.(67) receives no renormalization, provided that the coefficient b is replaced by $\beta(\alpha_s)/\alpha_s^2$ with $\beta(\alpha_s) = b\alpha_s^2 + O(\alpha_s^3)$ being the full beta function in QCD. This modification however only affects the overall normalization of the trace part, and can in fact be absorbed into the definition of the heavy quarkonium amplitudes. On the contrary, the relative coefficient of the traceless term in Eq.(67), i.e. the parameter κ , does depend on the normalization scale, which scale is appropriate to be chosen as the characteristic size of the heavy quarkonium [93]. However, given other uncertainties in the analysis of the two-pion transitions, the slow logarithmic variation of κ is a small effect.

5.1.2 Production of η by Gluonic Operators

The amplitude of production of the η meson by the quadratic gluonic operator is described by only one form factor: $\langle \eta | G_{\mu\nu} G_{\lambda\sigma} | 0 \rangle = A \epsilon_{\mu\nu\lambda\sigma}$ and is therefore entirely determined by the expression[98, 99] following from the chiral algebra and the anomaly in the divergence of the singlet axial current in QCD,

$$\epsilon^{\mu\nu\lambda\sigma} \langle \eta | G_{\mu\nu} G_{\lambda\sigma} | 0 \rangle = 16\pi^2 \sqrt{\frac{2}{3}} F_\eta m_\eta^2 , \quad (68)$$

where F_η is the η ‘decay constant’, equal to the pion decay constant $F_\pi \approx 130 \text{ MeV}$ in the limit of exact flavor SU(3) symmetry, and F_η is likely to be larger due to effects of the SU(3) violation.

A more interesting case is presented by the matrix elements relevant to discussion of transitions in quarkonium involving the $M2$ interaction from Eq.(57) and containing an extra covariant derivative: $\langle \eta(p) | G_{\mu\nu} D_\rho G_{\lambda\sigma} | 0 \rangle$ and $\langle \eta | (D_\rho G_{\mu\nu})^a G_{\lambda\sigma}^a | 0 \rangle$. It turns out that these matrix elements are also entirely determined by the relation (68). The possibility of the reduction of the structures with extra derivative to the expression in Eq.(68) follows from the general theory[105]. The reasoning in this specific case[100] makes use of the following identity, valid for arbitrary four-vector p :

$$p_\rho \epsilon_{\mu\nu\lambda\sigma} = p_\lambda \epsilon_{\mu\nu\rho\sigma} - p_\sigma \epsilon_{\mu\nu\rho\lambda} - p_\mu \epsilon_{\nu\rho\lambda\sigma} + p_\nu \epsilon_{\mu\rho\lambda\sigma} , \quad (69)$$

where the convention $\epsilon_{0123} = 1$ is assumed. The antisymmetry of the field tensor $G_{\mu\nu}$ then allows one to write the general form of the first of the discussed matrix elements in the linear order in the η momentum p in terms of two scalars X and Y :

$$i \langle \eta(p) | G_{\mu\nu}^a (D_\rho G_{\lambda\sigma})^a | 0 \rangle = X p_\rho \epsilon_{\mu\nu\lambda\sigma} + Y (p_\lambda \epsilon_{\mu\nu\rho\sigma} - p_\sigma \epsilon_{\mu\nu\rho\lambda}) . \quad (70)$$

The third structure, allowed by the symmetry and proportional to $(p_\mu \epsilon_{\nu\rho\lambda\sigma} - p_\nu \epsilon_{\mu\rho\lambda\sigma})$, is reduced to the first two due to the identity (69). Furthermore, applying in Eq.(70) the equations of motion (the Jacobi identity): $D_\rho G_{\lambda\sigma} + D_\sigma G_{\rho\lambda} + D_\lambda G_{\sigma\rho} = 0$, one arrives at the relation $X = 2Y$.

Likewise, writing the second of the discussed matrix elements in terms of two scalars \tilde{X} and \tilde{Y} as

$$i \langle \eta(p) | (D_\rho G_{\mu\nu})^a G_{\lambda\sigma}^a | 0 \rangle = \tilde{X} p_\rho \epsilon_{\mu\nu\lambda\sigma} + \tilde{Y} (p_\mu \epsilon_{\lambda\sigma\rho\nu} - p_\nu \epsilon_{\lambda\sigma\rho\mu}) , \quad (71)$$

and applying the Jacobi identity, one finds the relation $\tilde{X} = 2\tilde{Y}$.

The sum of the expressions (70) and (71) should combine into a total derivative, i.e. the sum should be proportional to p_ρ . This is possible due to the identity (69) under the condition that $\tilde{Y} = Y$, so that all the considered scalar form factors are expressed in terms of one of them, e.g. in terms of X :⁵

$$\tilde{X} = X, \quad \tilde{Y} = Y = \frac{1}{2} X . \quad (72)$$

Using this relation and contracting the sum of the expressions (70) and (71) with $\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}$ the form factor X is identified from the equation (68) as

$$X = -\frac{1}{60} \epsilon^{\mu\nu\lambda\sigma} \langle \eta | G_{\mu\nu} G_{\lambda\sigma} | 0 \rangle = -\frac{4\pi^2}{15} \sqrt{\frac{2}{3}} F_\eta m_\eta^2 . \quad (73)$$

5.1.3 Production of Single π^0 by Gluonic Operators

The production of a single neutral pion by gluonic operators obviously requires a breaking of the isotopic symmetry. Within the chiral approach the isospin breaking originates in the different masses of the up and down quarks, $m_u \neq m_d$. Allowing for this mass difference, the gluonic matrix elements for a single π^0 can be considered in essentially the same way as those for the η meson. In particular the ‘master equation’ for the anomalous amplitude in the case of pion takes the form[98]

$$\epsilon^{\mu\nu\lambda\sigma} \langle \eta | G_{\mu\nu} G_{\lambda\sigma} | 0 \rangle = 16\pi^2 \sqrt{2} \frac{m_d - m_u}{m_d + m_u} F_\pi m_\pi^2 . \quad (74)$$

Thus the simple rule of conversion from the amplitudes of the processes with η emission to similar processes with single π^0 can be done by replacing the coefficient $F_\eta m_\eta^2 \rightarrow \sqrt{3} F_\pi m_\pi^2 (m_d - m_u)/(m_d + m_u)$ and replacing the η momentum with that of the pion, $p_\eta \rightarrow p_\pi$. Naturally, these replacement rules can be also viewed in terms of a fixed $\eta - \pi$ mixing.

5.2 Two-pion Transitions

5.2.1 The Transition $\psi' \rightarrow \pi\pi J/\psi$

The general soft-pion relations of the chiral algebra require that the amplitude of a two-pion transition is bilinear in the components of the four-momenta of the pions[106, 107] in the chiral limit. For the

⁵An alternative derivation of two of these relations, namely $\tilde{X} = X$ and $\tilde{Y} = Y$, would be by arguing that in the particular amplitudes (70) and (71) the operators G^a and $(DG)^a$ can be considered as commuting with each other, so that the expressions (70) and (71) differ only by re-labeling the indices.

most studied transition $\psi' \rightarrow \pi\pi J/\psi$ this implies that in the limit of soft pions the amplitude can be generally parametrized as[106]

$$A(\psi' \rightarrow \pi^+\pi^- J/\psi) = [A(q^2 - 2m_\pi^2) + \lambda m_\pi^2] (\epsilon_1 \cdot \epsilon_2) + B E_1 E_2 (\epsilon_1 \cdot \epsilon_2) + C [(p_1 \cdot \epsilon_1)(p_2 \cdot \epsilon_2) + (p_2 \cdot \epsilon_1)(p_1 \cdot \epsilon_2)] , \quad (75)$$

where ϵ_1^μ and ϵ_2^μ are the polarization amplitudes of the initial and the final vector resonances, p_1 and p_2 are the 4-momenta of the two pions, E_1 and E_2 are their energies in the rest frame of the initial state, and $q = p_1 + p_2$ is the total 4-momentum of the dipion, so that $q^2 = m_{\pi\pi}^2$. Finally, A , B , C , and λ are the form factors, which should be considered constant in the soft pion limit, but are generally functions of kinematic variables beyond this limit. The spin-dependent term, proportional to C should be suppressed inasmuch as the charmed quark can be considered as heavy, since the spin-dependent interaction is proportional to the inverse of the heavy quark mass. The remaining constants in Eq.(75) can be related to the parameters in the leading order of the multipole expansion in QCD.

The two-pion transition between 3S_1 states is generated in the second order in the leading $E1$ term (Eq.(57) of the multipole expansion, so that the amplitude of the process can be written as

$$A(\psi' \rightarrow \pi^+\pi^- J/\psi) = \frac{1}{2} \langle \pi^+\pi^- | E_i^a E_j^a | 0 \rangle \alpha_{ij}^{(12)} , \quad (76)$$

where $\alpha_{ij}^{(12)}$ can be termed, in complete analogy with the atomic properties in electric field, as the transitional chromo-polarizability of the quarkonium. In other words, the $\psi_2 \rightarrow \psi_1$ transition in the chromo-electric field is described by the effective Hamiltonian

$$H_{eff} = -\frac{1}{2} \alpha_{ij}^{(12)} E_i^a E_j^a , \quad (77)$$

with the chromo-polarizability given by

$$\alpha_{ij}^{(12)} = \frac{1}{16} \langle 1S | \xi^a r_i \mathcal{G} r_j \xi^a | 2S \rangle , \quad (78)$$

where \mathcal{G} is the Green's function of the heavy quark pair in a color octet state. The latter function is not well understood presently, so that an *ab initio* calculation of the chromo-polarizability would be at least highly model dependent. Generally α_{ij} is a symmetric tensor. In the leading nonrelativistic order the coordinate and spin degrees of freedom factorize, so that the discussed chromo-polarizability for a transition between S wave states is actually reduced to a scalar:

$$\alpha_{ij}^{(12)} = \alpha^{(12)} \delta_{ij} (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2) . \quad (79)$$

The matrix element for production of the pion pair by the quadratic chromoelectric operator is then found from the general expression (67), and the result for the amplitude of the decay $\psi' \rightarrow \pi^+\pi^- J/\psi$ is[92, 93, 108]

$$A(\psi' \rightarrow \pi^+\pi^- J/\psi) = -\frac{4\pi^2}{b} \alpha^{(12)} \left[(q^2 + m_\pi^2) - \kappa \left(1 + \frac{2m_\pi^2}{q^2} \right) \left(\frac{(q \cdot P)^2}{P^2} - \frac{1}{4} q^2 \right) + \frac{3\kappa}{2} \frac{\ell_{\mu\nu} P^\mu P^\nu}{P^2} \right] (\epsilon_1 \cdot \epsilon_2) , \quad (80)$$

where P is the 4-momentum of the initial ψ' resonance, and the tensor $\ell_{\mu\nu}$ corresponds to a D wave motion in the c.m. frame of the dipion and is written in terms of the relative momentum $r = p_1 - p_2$ of the two pions as

$$\ell_{\mu\nu} = r_\mu r_\nu + \frac{1}{3} \left(1 - \frac{4m_\pi^2}{q^2} \right) (q^2 g_{\mu\nu} - q_\mu q_\nu) . \quad (81)$$

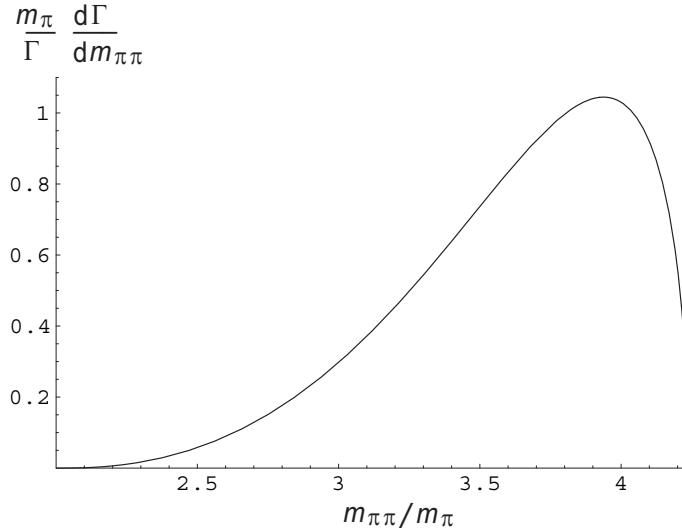


Figure 4: The spectrum of the dipion invariant masses $m_{\pi\pi}$ in the decay $\psi' \rightarrow \pi^+\pi^- J/\psi$ as given by the amplitude (80) with $\kappa = 0.2$.

The constants in the parametrization (75) can be thus found as

$$A = -\frac{4\pi^2}{b} \alpha^{(12)} \left(1 + \frac{3}{4}\kappa\right), \quad B = 6\kappa \frac{4\pi^2}{b} \alpha^{(12)}, \quad \lambda = \frac{12\kappa}{4 + 3\kappa}, \quad (82)$$

while the parameter C is clearly equal to zero, since the $E1$ interaction carries no spin dependence. It can be also noted that Eq.(80) differs from (75) at $C = 0$ in that it contains only two parameters, while the third, λ is fully fixed as shown in Eq.(82). This is due to the chiral constraints and the conformal anomaly (Eq.(60)) fully determining the terms of order m_π^2 relative to those quadratic in the pion momenta.

One can readily see from Eq.(80) that the overall rate of the decay is determined by the chromo-polarizability $\alpha^{(12)}$, while the shape of the spectrum depends on the parameter κ . Furthermore, the last term in the braces in Eq.(80) is proportional to κ and describes the D wave motion in the c.m. frame of the two pions, which D wave correlates with the motion of the dipion as a whole in the rest frame of the decaying ψ' . Thus the formula (80) based on the leading $E1$ term of the multipole expansion predicts[93] a relation between the amplitude of the D wave and the parameter in the shape of the spectrum in the decay. The most detailed to date experimental study of the decay $\psi' \rightarrow \pi^+\pi^- J/\psi$ using this formula was done by BES[109]. The fit to the spectrum of the invariant mass of the produced dipion resulted in the value $\kappa = 0.186 \pm 0.003 \pm 0.006$, while the fit to the ratio of the D and S wave amplitudes from the angular distribution gave $\kappa = 0.210 \pm 0.027 \pm 0.042$. Clearly, the consistency of these two values implies that the discussed approach correctly predicts the ratio of the D wave in terms of the sub-dominant term proportional to κ in the S wave amplitude. It can be also noted that at $\kappa \approx 0.2$ the contribution of the D wave to the total rate is quite small – only about 2% [109]. The familiar characteristic shape of the spectrum of the two-pion invariant masses is shown at $\kappa = 0.2$ in Fig.4. Comparing the integral over this spectrum with the experimental total decay rate one can evaluate the transitional chromo-polarizability as $\alpha^{(12)} \approx 2 \text{ GeV}^{-3}$.

It can be also noted that in the discussed picture the pions are produced in pure isoscalar $I = 0$ state, which implies fixed relation between the differential rates of emission of pairs of neutral and of charged pions $d\Gamma(\psi' \rightarrow \pi^0\pi^0 J/\psi) = 0.5d\Gamma(\psi' \rightarrow \pi^+\pi^- J/\psi)$. The most precise test of this relation has been done by the CLEO-c experiment[110], which resulted in the measured ratio of the total decay

rates $0.4924 \pm 0.0047 \pm 0.0086$.

5.2.2 Effects of the Final State Interaction Between Pions

So far the amplitude of the two-pion transition was considered in the chiral limit. The formula in Eq.(67) is exact in the leading chiral order, i.e. as far as only the quadratic terms in the pion momenta and mass are concerned. In particular this expression receives no corrections due to the final state interaction (FSI) between the pions. The latter interaction however can give rise to the terms whose expansion starts with the fourth power of momenta and the pion mass, and generally can modify the amplitude at momenta of the pions relevant for actual transitions in quarkonium. The effects of FSI in chiral treatment of the two-pion transitions in quarkonium were a matter of concern ever since the earlier theoretical analyses [106]. The effect in the phases of the amplitudes is well known: these phases for the production amplitudes are equal to the two-pion scattering phases in the corresponding partial waves: $S = |S| \exp(i\delta_0)$, $D = |D| \exp(i\delta_2)$, where the $I = 0$ phases for the S wave, δ_0 , and for the D wave, δ_2 are quite well studied.⁶ It is also generally estimated both on theoretical and phenomenological grounds that the FSI corrections are not big (at most 20 - 25%) in the transitions $\psi' \rightarrow \pi\pi J/\psi$ and $\Upsilon' \rightarrow \pi\pi\Upsilon$. (For a discussion see the review [111].) Some phenomenological arguments in favor of such estimate will also be discussed here after presenting a theoretical estimate of the onset of the higher term in the chiral expansion.

The interaction of pions at low energy in the D wave is quite weak, so that any modification by FSI of the D wave production amplitude can be safely neglected, and only the modification of the S wave amplitude is of interest for present phenomenology. The imaginary part of the correction at $q^2 > 4m^2$ is found from the unitarity relation in terms of the isospin $I=0$ $\pi\pi \rightarrow \pi\pi$ scattering amplitude $T(q^2)$ in the S wave as

$$\text{Im}(\delta S) = \sqrt{1 - \frac{4m_\pi^2}{q^2}} \frac{T(q^2)}{16\pi} S. \quad (83)$$

The amplitude $T(q^2)$ is well known in the chiral limit, i.e. in the quadratic in q and m_π approximation, since the work of Weinberg [112]. In the normalization used here this amplitude has the form

$$T(q^2) = \frac{2q^2 - m_\pi^2}{F_\pi^2}, \quad (84)$$

where $F_\pi \approx 130$ MeV is the $\pi^+ \rightarrow \mu^+\nu$ decay constant. Clearly, the expression in Eq.(83) is of the fourth power in q and m .

The real part of the correction to the S wave production amplitude δS can then be estimated from Eq.(83) using the dispersion relation in q^2 for the amplitude S . In doing so one should set the condition for the subtraction constants that this real part does not contain quadratic (and certainly also constant) terms in q and m , since these are given by Eq.(67). After these subtractions the remaining dispersion integral is still logarithmically divergent and contains the well known ‘chiral logarithm’, depending on the ultraviolet cutoff Λ , which is usually set at $\Lambda \sim 1$ GeV, i.e. the scale where any chiral expansion certainly breaks down. Using this approach one can estimate[108] the corrections in the S wave part of the amplitude given by Eq.(80) as

$$A(\psi' \rightarrow \pi^+\pi^- J/\psi) = -\frac{4\pi^2}{b} \alpha^{(12)} \left[(q^2 + m_\pi^2)(1 + \xi_1) - \kappa \left(1 + \frac{2m_\pi^2}{q^2} \right) (1 + \xi_2) \left(\frac{(q \cdot P)^2}{P^2} - \frac{1}{4} q^2 \right) + \frac{3\kappa}{2} \frac{\ell_{\mu\nu} P^\mu P^\nu}{P^2} \right] (\epsilon_1 \cdot \epsilon_2), \quad (85)$$

⁶It can be mentioned that the analysis [109] of the data on the $\psi' \rightarrow \pi^+\pi^- J/\psi$ decay does not take into account the relative phase between the S and D wave pion production amplitudes. Thus it would be interesting to know whether including the phase factor in the angular analysis, produces a significant impact on the results.

where the correction terms ξ_1 and ξ_2 are given by

$$\xi_1 = \frac{2(q^2)^2 - 7q^2 m_\pi^2 + 3m_\pi^4}{16\pi^2 F_\pi^2 (q^2 + m_\pi^2)} \ln \frac{\Lambda^2}{m_\pi^2} + i \frac{2q^2 - m_\pi^2}{16\pi F_\pi^2} \sqrt{1 - \frac{4m_\pi^2}{q^2}}, \quad (86)$$

and

$$\xi_2 = \frac{2(q^2)^2 - 9q^2 m_\pi^2 + 8m_\pi^4}{16\pi^2 F_\pi^2 (q^2 + 2m_\pi^2)} \ln \frac{\Lambda^2}{m_\pi^2} + i \frac{2q^2 - m_\pi^2}{16\pi F_\pi^2} \sqrt{1 - \frac{4m_\pi^2}{q^2}}, \quad (87)$$

where the non-logarithmic imaginary part is retained for reference regarding the normalization. The lower limit under the logarithm is generally a function of both q^2 and m_π^2 , however any difference of this function from the value m^2 used in Eqs.(86) and (87) is a non-logarithmic term, i.e. beyond the accuracy of these equations. Since m_π^2 is the smallest of the two parameters in the physical region of pion production, it can be expected that using this parameter provides a conservative estimate of the effect of FSI.

Estimating the corrections in Eq.(86) and Eq.(87), one finds that at the lower end of the physical phase space, i.e. near $q^2 = 4m_\pi^2$, these terms do not exceed few percent. Thus the corrections only weakly modify the normalization of the pion production amplitude near the threshold. A theoretical extrapolation to higher values of q^2 is problematic, and, most likely, one would have to resort to using actual data on the dipion spectra in order to judge upon the significance of deviation from the essentially linear in q^2 behavior of the amplitude described by Eq.(80). A quantitative estimate of the deviation from this behavior has been attempted [111] using the data [113] on $\Upsilon' \rightarrow \pi^+\pi^-\Upsilon$ by parametrizing the deviation as a factor $(1 + q^2/M^2)$ in the amplitude with M being a parameter. The thus obtained lower limit on M is 1 GeV at 90% C.L.

Another phenomenological argument in favor of a relatively moderate FSI effect in the absolute value of the dominant S wave in the decay $\psi' \rightarrow \pi^+\pi^-J/\psi$ stems from the previously mentioned agreement of the observed [109] value of the ratio D/S with the parameter κ entering the expression for the S wave and extracted from the two-pion invariant mass spectrum. Clearly such an agreement would be ruined if there was a significant enhancement of the S wave by FSI.

The treatment of the FSI effects in the two-pion transition amplitude in fact reveals a certain inadequacy of the parametrization in Eq.(75). Indeed the factors B and C are each contributed by both the S and D wave motion in the c.m. frame of the two pions, and those contributions are differently modified by the FSI effects. In this situation it is more reasonable to use the parametrization[108, 114] in terms of separate partial waves:

$$A(\psi' \rightarrow \pi^+\pi^-J/\psi) = S(\epsilon_1 \cdot \epsilon_2) + D_1 \ell_{\mu\nu} \frac{P^\mu P^\nu}{P^2} (\epsilon_1 \cdot \epsilon_2) + D_2 q_\mu q_\nu \epsilon^{\mu\nu} + D_3 \ell_{\mu\nu} \epsilon^{\mu\nu}, \quad (88)$$

where $\epsilon^{\mu\nu}$ stands for the spin-2 tensor made from the polarization amplitudes of the 3S_1 resonances

$$\epsilon^{\mu\nu} = \epsilon_1^\mu \epsilon_2^\nu + \epsilon_1^\nu \epsilon_2^\mu + \frac{2}{3} (\epsilon_1 \cdot \epsilon_2) \left(\frac{P^\mu P^\nu}{P^2} - g_{\mu\nu} \right). \quad (89)$$

The terms in the expression (88) describe an S wave and three possible types of D -wave motion: the term with D_1 corresponds to a D wave in the c.m. system of the two pions correlated with the overall motion of the dipion in the rest frame of the initial state, the D_2 term describes the D -wave motion of the dipion as a whole, correlated with the spins of the quarkonium resonances, and finally, the D_3 term corresponds to the correlation between the spins and the D -wave motion in the c.m. frame of the dipion. Clearly, the factors S and D_2 are modified by the FSI in the S -wave dipion, while the factors D_1 and D_3 receive a modification from the two-pion D wave FSI. The partial-wave form factors are

expressed in terms of the parameters in Eq.(75) as

$$\begin{aligned} S &= \left(A + \frac{1}{3} C \right) (q^2 - 2 m_\pi^2) + \lambda m_\pi^2 + \frac{1}{12} \left(B - \frac{2}{3} C \right) \left[3 q_0^2 - (q_0^2 - q^2) \left(1 - \frac{4m_\pi^2}{q^2} \right) \right] \\ D_1 &= -\frac{1}{4} \left(B - \frac{2}{3} C \right), \quad D_2 = \frac{1}{6} C \left(1 + \frac{2m_\pi^2}{q^2} \right), \quad D_3 = -\frac{1}{4} C. \end{aligned} \quad (90)$$

Naturally, in the presented treatment, based on the multipole expansion in the leading nonrelativistic order, only the spin-independent amplitudes S and D_1 are present, as described by Eq.(80) and Eq.(85).

5.3 Single η and Single Pion Transitions

5.3.1 $\psi' \rightarrow \eta J/\psi$

The decay $\psi' \rightarrow \eta J/\psi$ arises due to the interference of the $E1$ and $M2$ terms in the multipole expansion (Eq.(57)). The amplitude of the transition can thus be written as

$$A(\psi' \rightarrow \eta J/\psi) = m_Q^{-1} \langle \eta | E_i^a (D_j B_k)^a + (D_j B_k)^a E_i^a | 0 \rangle A_{ijk}, \quad (91)$$

where

$$A_{ijk} = \frac{1}{64} \langle 2^3 S_1 | \xi^a r_i \mathcal{G} r_j \xi^a S_k | 1^3 S_1 \rangle \quad (92)$$

is the quarkonium transition amplitude. In the approximation of factorized coordinate and spin degrees of freedom the matrix element of the spin operator is expressed in terms of the polarization amplitudes $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ of the initial and final quarkonium resonances, while the remaining coordinate overlap is the same as in the expression (78) for the chromo-polarizability, so that the amplitude is expressed in terms of $\alpha^{(12)}$ as

$$A(\psi' \rightarrow \eta J/\psi) = i \frac{\alpha^{(12)}}{4 m_Q} \langle \eta | E_i^a (D_i B_k)^a + (D_i B_k)^a E_i^a | 0 \rangle \epsilon_{klm} \epsilon_{1l} \epsilon_{2m}, \quad (93)$$

The amplitude of the η production by the gluonic operator in this expression is readily found from the relations (70) - (73), and is given by

$$i \langle \eta | E_i^a (D_i B_k)^a + (D_i B_k)^a E_i^a | 0 \rangle = \frac{16 \pi^2}{15} \sqrt{\frac{2}{3}} F_\eta m_\eta^2 p_k \quad (94)$$

with \vec{p} being the momentum of the η meson. As a result one finds the following simple formula for the amplitude of the η transition

$$A(\psi' \rightarrow \eta J/\psi) = \frac{4 \pi^2}{15} \sqrt{\frac{2}{3}} \frac{\alpha^{(12)}}{m_Q} F_\eta m_\eta^2 \epsilon_{klm} p_k \epsilon_{1l} \epsilon_{2m}. \quad (95)$$

Comparing the latter formula with Eq.(80) one finds that the quarkonium part of the amplitude, $\alpha^{(12)}$, cancels in the ratio of the amplitudes for the η and the two-pion emission and the ratio is essentially determined by the two anomalies in QCD: the conformal anomaly and the one in the divergence of the singlet axial current[92]. This remarkable relation can be written in terms of the decay rates as

$$\frac{\Gamma(\psi' \rightarrow \eta J/\psi)}{d\Gamma(\psi' \rightarrow \pi^+ \pi^- J/\psi)/dq^2} = \frac{64 \pi^2}{25} \frac{F_\eta^2}{m_c^2} \frac{p_\eta^3}{|\vec{q}|} \left(\frac{m_\eta^2}{q^2} \right)^2 \left(1 - \frac{4m_\pi^2}{q^2} \right)^{-1/2} \frac{1}{\mathcal{F}}, \quad (96)$$

where the factor \mathcal{F} describes the deviation of the two-pion transition amplitude from the limit of dominance of the anomaly contribution ($\kappa \rightarrow 0$) and of massless pion:

$$\mathcal{F} = \left| 1 + \frac{m_\pi^2}{q^2} - \frac{\kappa}{q^2} \left(1 + \frac{2m_\pi^2}{q^2} \right) \left[\frac{(q \cdot P)^2}{P^2} - \frac{1}{4} q^2 \right] \right|^2 + \frac{\kappa^2}{5} \left[\frac{(q \cdot P)^2}{q^2 P^2} - 1 \right]^2 \left(1 - \frac{4m_\pi^2}{q^2} \right)^2, \quad (97)$$

where the last term describes the small contribution of the D wave. Using $\kappa = 0.2$ and integrating over the phase space of the two-pion transition one can arrive at the estimate of the ratio of the total rates:

$$\frac{\Gamma(\psi' \rightarrow \eta J/\psi)}{\Gamma(\psi' \rightarrow \pi^+ \pi^- J/\psi)} = 0.09 \left(\frac{F_\eta}{130 \text{ MeV}} \right)^2 \left(\frac{1.4 \text{ GeV}}{m_c} \right)^2. \quad (98)$$

Given all the uncertainties, this estimate agrees very well with experimental value 0.097 ± 0.003 for this ratio.

It can be mentioned as a sidenote, that being applied to the transitions in bottomonium between Υ' and Υ , Eq.(96), using the appropriate for these transitions value $\kappa \approx 0.15$, predicts the ratio of the rates

$$\frac{\Gamma(\Upsilon' \rightarrow \eta \Upsilon)}{\Gamma(\Upsilon' \rightarrow \pi^+ \pi^- \Upsilon)} = (2.2 \times 10^{-3}) \left(\frac{F_\eta}{130 \text{ MeV}} \right)^2 \left(\frac{4.8 \text{ GeV}}{m_b} \right)^2, \quad (99)$$

corresponding to $\mathcal{B}(\Upsilon' \rightarrow \eta \Upsilon) \approx 4.3 \times 10^{-4}$. This prediction can be compared with the recent preliminary data from CLEO[115]: $\mathcal{B}(\Upsilon' \rightarrow \eta \Upsilon) = (2.5 \pm 0.7 \pm 0.5) \times 10^{-4}$.

Another sidenote regarding the decay $\psi' \rightarrow \eta J/\psi$ is that recently this process has been used not for its own sake, but rather as a precision source of the η mesons for high accuracy measurements of the η mass[116] and its decays[117].

5.3.2 $\psi' \rightarrow \pi^0 J/\psi$

The decay $\psi' \rightarrow \pi^0 J/\psi$ requires breaking of the isotopic symmetry. If the relevant to this process breaking is due to the mass difference between the u and d quarks the amplitude of this decay can be found from the amplitude of the η transition by applying the previously discussed conversion factor, so that the ratio of the rates for these two transitions can be estimated[118] as

$$\frac{\Gamma(\psi' \rightarrow \pi^0 J/\psi)}{\Gamma(\psi' \rightarrow \eta J/\psi)} = 3 \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 \frac{F_\pi^2}{F_\eta^2} \frac{m_\pi^4}{m_\eta^4} \frac{p_\pi^3}{p_\eta^3}. \quad (100)$$

In the limit of the flavor SU(3) symmetry one has $F_\pi = F_\eta$. In reality it is known from comparison of F_π and F_K that the presence of heavier strange quarks increases the constant F , so that F_η is expected to be larger than F_π . Therefore the limit $F_\pi = F_\eta$ can be used as an upper bound on the ratio of the rates in Eq.(100). The ratio of the masses of the u and d quarks, describing the explicit breaking of the chiral symmetry and the isospin violation in this breaking was studied years ago in great detail by Gasser and Leutwyler[119]. The largest value for the ratio $(m_d - m_u)/(m_d + m_u)$ allowed by that study is approximately 0.35. Thus the theoretical upper bound for the ratio of the decay rates in Eq.(100) is approximately 2.3%, which is still by more than 4σ below the experimental result[110]: $(4.1 \pm 0.4 \pm 0.1)\%$. It can be also mentioned in connection with the light quark mass ratio that the well known Weinberg's formula[120] gives

$$\frac{m_d - m_u}{m_d + m_u} = \frac{m_{K^0}^2 - m_{K^+}^2 + m_{\pi^+}^2 - m_{\pi^0}^2}{m_{\pi^0}^2} = 0.285, \quad (101)$$

and results in a still smaller ratio of the decay rates if used in Eq.(100). It is certainly understood[118] that the formula (100) may receive unexpectedly large corrections from the effects of the SU(3) violation, however such corrections can also significantly affect the analysis of the chiral phenomenology in Ref.[119], and the whole subject then would have to be revisited anew. It should be mentioned that the largest theoretical estimate of the discussed ratio of the transition rates found in the literature[121] corresponds to 3.4%, which is also significantly lower than the experimental number. However, the latter estimate does not fully take into account the proper QCD structure of the relevant amplitude for the meson production by the gluonic operator.

It therefore looks like the isospin violation by the light quark masses is not sufficient to describe the data[110] and one has to assume that at least one of the charmonium states in the transition is in fact not a pure isoscalar, but contains a small admixture of an isovector $I = 1$ four-quark state. We shall further discuss such possibility in the Section 6.1.2.

5.3.3 $\psi' \rightarrow \pi^0 h_c$ and $h_c \rightarrow \pi^0 J/\psi$

The transitions between the 3S_1 and 1P_1 states with emission of π^0 arise in the discussed picture through the interference of the $E1$ and $M1$ interaction terms in Eq.(57). The pion is emitted in the S wave, and the amplitude for the transition can be written as

$$A({}^3S_1 \rightarrow \pi^0 {}^3P_1) = \langle \pi^0 | E_i^a B_k^a | 0 \rangle I \epsilon_{1k} \epsilon_{2i} , \quad (102)$$

where I is the quarkonium radial overlap integral

$$I = -\frac{1}{96 m_Q} \langle P | \xi^a (r \mathcal{G}_S + \mathcal{G}_P r) \xi^a | S \rangle \quad (103)$$

with \mathcal{G}_S and \mathcal{G}_P being the octet-state Green's function in the corresponding partial waves.

The gluonic matrix element for the single pion production is readily found from Eq.(74), and one finds the following expression for the decay rate[122]

$$\Gamma({}^3S_1 \rightarrow \pi^0 {}^3P_1) = \left(\frac{2\pi^2}{3} \frac{m_d - m_u}{m_d + m_u} F_\pi m_\pi^2 \right)^2 |I|^2 \frac{p_\pi}{\pi} . \quad (104)$$

A numerical evaluation of the decay rate for the transition $\psi' \rightarrow \pi^0 h_c$ greatly suffers from a poor knowledge of the overlap integral in Eq.(103). If, for an order-of-magnitude estimate, one uses typical r as in the radiative transitions, $r \sim 0.4$ fm, and also approximates $\mathcal{G} \sim 1 \text{ GeV}^{-1}$, then one very approximately estimates $|I| \sim 0.15 \text{ GeV}^{-3}$, and also estimates the rate from Eq.(104) as $\Gamma(\psi' \rightarrow \pi^0 h_c) \sim 15 \text{ eV}$, which corresponds to the branching fraction of only about 5×10^{-5} . The latter number is by an order of magnitude smaller than the measured[30] combined branching fraction $\mathcal{B}(\psi' \rightarrow \pi^0 h_c) \times \mathcal{B}(h_c \rightarrow \gamma \eta_c) = (4.0 \pm 0.8 \pm 0.7) \times 10^{-4}$. It is not clear whether the estimate of the overlap I is totally off the mark, or the apparently enhanced decay $\psi' \rightarrow \pi^0 h_c$ also proceeds due to a small four-quark admixture in the ψ' .

A similar estimate of at most few tens eV is applicable to the transition from the h_c resonance, $h_c \rightarrow \pi^0 J/\psi$. The experimental status of this decay is still not clear. An early observation of this decay in the E760 experiment[28] has not been confirmed by the E835 data[29] a decade later. Clearly, an additional experimental input on this decay would be of a great interest.

5.4 Chromo-polarizability and Slow Charmonium in Matter

5.4.1 Diagonal Chromo-polarizability of J/ψ and ψ'

As discussed, the hadronic transitions from ψ' to J/ψ are determined by the transitional chromo-polarizability $\alpha^{(12)}$, defined in Eq.(78). Similar diagonal quantities $\alpha^{(11)} = \alpha_\psi$ and $\alpha^{(22)} = \alpha_{\psi'}$ determine the interaction of the J/ψ and ψ' resonances with soft gluonic fields and are of a great importance for description of a number of processes including the interaction of the charmonium resonances with nuclear matter.

At present these characteristics of the charmonium resonances are not well known. An early calculation[123] of the chromo-polarizability of quarkonium ground state relied on the treatment of the system as being purely Coulomb-like, which is a valid limit for very heavy quarkonium, but which

hardly can be applied to charmonium. One guidance for the value of the discussed parameters is set by the transitional chromo-polarizability $\alpha^{(12)} \approx 2 \text{ GeV}^{-3}$. Namely the diagonal terms α_ψ and $\alpha_{\psi'}$ should satisfy the Schwartz inequality

$$\alpha_\psi \alpha_{\psi'} \geq (\alpha^{(12)})^2 , \quad (105)$$

and they both should be real and positive since there are no intermediate states that would enter the second-order correlator in the $E1$ interaction over either of the vector resonances. Thus it is reasonable to consider the value of $\alpha^{(12)}$ as a ‘reference’ benchmark for either of the diagonal terms.

5.4.2 The Decay $J/\psi \rightarrow \pi\pi\ell^+\ell^-$

The chromo-polarizability of the J/ψ resonance can in fact be measured experimentally in the decay $J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-$ with a soft pion pair[124]. Using the same description based on the QCD multipole expansion as for the two-pion transitions, one can write the amplitude of the decay in the form

$$A(J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-) = \frac{1}{2} \langle \pi^+\pi^- | (\vec{E}^a)^2 | 0 \rangle \sum_{n=1} \frac{\alpha^{(1n)}}{m(nS) - m_{J/\psi} + q_0} A(n^3S_1 \rightarrow \ell^+\ell^-) , \quad (106)$$

where the sum goes over the discrete states as well as the continuum, and $q_0 = (q \cdot P)/m_{J/\psi}$. In writing this expression it is taken into account that the soft pion approximation is only valid at $q_0 \ll m_{J/\psi}$, so that any recoil of the heavy quarkonium upon emission of the pion pair can be and is neglected. In this limit the relation between q_0 and the total momentum l of the lepton pair can also be written as $m_{J/\psi}^2 - l^2 = 2q_0 m_{J/\psi}$.

In the chiral limit the first term (with $n = 1$) in the sum in Eq.(106) dominates for soft pions, due to its singular behavior as $1/q_0$. It can be noticed however that the decay amplitude itself is not singular due to the (even faster) vanishing of the pion production amplitude (67) in the limit of soft pions. In the ‘real life’ the minimal practical energy q_0 is not much less than the spacing of the quarkonium levels, and the contribution of higher states mixes into the amplitude. In what follows we first consider the contribution of only the first term of the sum in Eq.(106) and then discuss the effect of the higher terms. Keeping only the contribution of the first term in the sum in Eq.(106), one can write the differential rate of the discussed decay in terms of the chromo-polarizability α_ψ and the leptonic width $\Gamma_{ee}(J/\psi)$ in the form

$$d\Gamma(J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-) = \frac{(q^2)^2 \mathcal{F}}{4b^2 q_0^2} |\alpha_\psi|^2 \sqrt{1 - \frac{4m_\pi^2}{q^2}} \sqrt{q_0^2 - q^2} \Gamma_{ee}(J/\psi) dq^2 dq_0 , \quad (107)$$

where the factor \mathcal{F} is the same as defined in Eq.(97).

In order to assess the feasibility of observing the discussed decays it can be noted that at a given constraint on the maximal value of q_0 : $q_0 < \Delta$ (or equivalently at a lower cutoff on the invariant mass of the lepton pair) the probability described by Eq.(107) strongly peaks near the highest values of both q^2 and q_0 , i.e. $q^2 \sim \Delta^2$ and $q_0 \sim \Delta$, and the total probability in the kinematical region constrained as $q_0 < \Delta$ scales approximately as Δ^6 . However at higher q^2 both the dominance of the diagonal $1S - 1S$ transition in the sum in Eq.(106) becomes weaker and the behavior of the amplitude in Eq.(67) derived for soft pions becomes questionable. It is still quite likely that with these limitations the discussed here approach can be used up to somewhat higher values of Δ : $\Delta \approx 0.8 - 0.9 \text{ GeV}$, than those observed in the pionic transitions from ψ' and Υ' and that the $F_0(980)$ resonance places a natural upper bound on the region of applicability of Eq.(67). As to the contribution of higher quarkonium states in the sum in Eq.(106), for each of these states the magnitude of this contribution relative to that of the diagonal transition is given by

$$r_n = \left| \frac{\alpha^{(1n)}}{\alpha_\psi} \frac{q_0}{m_n - m_{J/\psi} + q_0} \right| \left[\frac{\Gamma_{ee}(n^3S_1)}{\Gamma_{ee}(J/\psi)} \right]^{1/2} . \quad (108)$$

The transition polarizability $\alpha^{(1n)}$ should considerably decrease with n . This is supported by the very small experimental rate of the decay $\Upsilon(3S) \rightarrow \pi\pi\Upsilon$ (a discussion of this decay in terms of the transitional chromo-polarizability can be found in Ref.[108]). Thus, most likely, the only real effect of higher states up to $\Delta \sim 0.9 \text{ GeV}$ reduces to that of the ψ' resonance. This effect however can be accounted for in the data analysis, since for this resonances all the parameters (except for the overall relative sign of its contribution) in Eq.(108) are known. Furthermore an observation and analysis of the discussed decays at higher values of q_0 , where the expression (67) is no longer valid, would be of a great interest for studies of the pion-pion scattering beyond the soft-pion region.

Numerically one can estimate from Eq.(107) the total rate in the kinematical region constrained by $\Delta = 0.9 \text{ GeV}$ as

$$\Gamma(J/\psi \rightarrow \pi^+\pi^-\ell^+\ell^-)|_{q_0 < 0.9 \text{ GeV}} \approx 10^{-4} \left| \frac{\alpha_\psi}{2 \text{ GeV}^{-3}} \right|^2 \Gamma_{ee}(J/\psi). \quad (109)$$

Given that the diagonal polarizability is likely to be larger than the transition one, it can be expected that for the J/ψ resonance the branching ratio of the discussed decay in a useable kinematical range should be at the level of 10^{-5} which looks to be well within the reach with the expected CLEO-c data sample.

5.4.3 Slow J/ψ in Nuclear Medium

Understanding the charmonium interaction with nuclear matter is important for description of the photo- and hadro- production of charmonium and charmed hadrons on the nuclear targets as well as for diagnostics of the hadronic final states in heavy-ion collisions and search for Quark Gluon Plasma. Such interaction has been a subject of numerous studies with a broad range of theoretical predictions.

First perturbative QCD calculations [125, 123] predicted very small J/ψ dissociation cross section by hadrons, on the order of few μbarn . With all the great interest to the problem of charmonium interaction with nucleons and nuclear matter and its practical importance, the discussion of this interaction is still wide open. In particular, the estimates of the strength of the interaction of J/ψ and ψ' with the nucleon range, in terms of the scattering cross section at low energy, from a fraction of millibarn [126, 127] up to 10 mb or more [128, 129, 130]. Recent reviews of the subject and further references can be found in the Refs. [131, 132, 133].

In many of these applications the most interesting energy region is usually well above the threshold, where the complexity of the problem becomes more confounding due to the multitude of possible inelastic processes contributing to charmonium scattering on nuclear matter. However the strength of the interaction at energy close to the threshold is also measurable [128] and its reliable estimate can serve as a useful reference point for analyses of the behavior of the interaction at higher energies. Furthermore, the J/ψ and ψ' interactions at low energies are of explicit importance for high energy heavy ion collisions since the relative motion between the co-moving charmonium and nuclear matter is rather slow. Moreover the forward elastic scattering amplitude can be related to the J/ψ and ψ' mass shift in matter predicted by a number of models [134, 135, 136, 137].

The interaction of a slow charmonium with nucleons can be considered within the multipole expansion in QCD in terms of the chromo-polarizability[127, 138], which parametrizes the interaction of charmonium with soft gluon fields inside the nucleons. For the J/ψ resonance the amplitude of elastic scattering on a nucleon is then given by

$$A(J/\psi N \rightarrow J/\psi N) = \frac{\alpha_\psi}{2} \langle N | (\vec{E}^a \cdot \vec{E}^a) | N \rangle. \quad (110)$$

It can be mentioned that at low energies, below the threshold for the process $J/\psi + N \rightarrow \Lambda_c + D$, the only kinematically allowed inelastic reaction is $J/\psi + N \rightarrow \eta_c + N$. However the latter reaction involves the heavy quark spin-flip and should be suppressed in comparison with the elastic scattering. In order

to evaluate the matrix element of the gluonic operator over the nucleon in Eq.(110) one can make use of the conformal anomaly relation (60). Namely one can write

$$\langle N | (\vec{E}^a \cdot \vec{E}^a) - (\vec{B}^a \cdot \vec{B}^a) | N \rangle = -\frac{1}{2} \langle N | (G_{\mu\nu}^a)^2 | N \rangle = \frac{16\pi^2}{9} \langle N | \theta_\mu^\mu | N \rangle = \frac{16\pi^2}{9} 2m_N^2 . \quad (111)$$

Given that the average of the operator $(\vec{B}^a \cdot \vec{B}^a)$ over nucleon is non-negative, one arrives at the inequality[138]

$$\langle N | (\vec{E}^a \cdot \vec{E}^a) | N \rangle \geq \frac{16\pi^2}{9} 2m_N^2 . \quad (112)$$

It is expected however[138] that the chromomagnetic contribution to the nucleon mass is substantially smaller than the chromo-electric one, so that the actual value of the average in Eq.(112) should be close to the lower bound.

The elastic $J/\psi N$ scattering amplitude in the low energy limit is thus estimated as

$$A(J/\psi N \rightarrow J\psi N) \geq \frac{16\pi^2}{9} \alpha_\psi m_N^2 \quad (113)$$

and the corresponding scattering length is then given by

$$a_{J/\psi N} = \frac{1}{4\pi} A(J/\psi N \rightarrow J\psi N) \frac{m_{J/\psi}}{m_{J/\psi} + m_N} \geq 0.37 \text{ fm} \left(\frac{\alpha_\psi}{2 \text{ GeV}^{-3}} \right) . \quad (114)$$

Accordingly, the elastic cross section at the threshold is found as

$$\sigma(J/\psi N \rightarrow J\psi N) = 4\pi a_{J/\psi N}^2 \geq 17 \text{ mb} \left(\frac{\alpha_\psi}{2 \text{ GeV}^{-3}} \right)^2 . \quad (115)$$

The positive sign of the scattering length implies attraction of the J/ψ to nucleons, and the strength of this attraction can be expressed in terms of the binding potential for J/ψ in nuclear matter with the number density of nucleons $\rho_N = 0.16 \text{ fm}^{-3}$:

$$V_{J\psi} = -A(J/\psi N \rightarrow J\psi N) \frac{\rho_N}{2m_N} \leq -21 \text{ MeV} \left(\frac{\alpha_\psi}{2 \text{ GeV}^{-3}} \right) . \quad (116)$$

Such rather strong attraction indicates a possibility of existence of bound states of J/ψ in light nuclei. Indeed, the condition for existence of a bound state in the approximation, where a nucleus is considered as being of a uniform density ρ_N up to the sharp boundary at the radius R_A reads as

$$R_A^2 > \frac{\pi^2}{8m_{J/\psi}(-V_{J\psi})} . \quad (117)$$

With the minimal estimate of the binding potential in Eq.(116) this condition is satisfied already at $R_A > 0.9 \text{ fm}$, which points to a relevance of the problem of bound states to light nuclei. Although the criterion in Eq.(117) is not directly applicable for light nuclei, the resulting estimate gives credibility to the claims [128, 139] that bound states of the J/ψ resonance in nuclei do exist starting from light nuclei. With regards to existence of a near-threshold bound or resonant state of the J/ψ and a single nucleon, the present understanding is generally insufficient for arriving at a definite conclusion.

5.4.4 Interaction of slow ψ' with Nucleons

The consideration of the elastic $\psi'N$ scattering amplitude parallels that for the $J/\psi N$ process with the obvious replacement of α_ψ by the chromo-polarizability $\alpha_{\psi'}$ of the ψ' . The latter parameter is expected to be larger than the ‘reference’ value of 2 GeV^{-3} , so that the numerical value of the ‘minimal’ binding energy in nuclear matter -21 MeV is very likely to be overly conservative. The shift of the mass of the ψ' resonance in nuclear matter can be important for consideration of the decay $\psi' \rightarrow D\bar{D}$ which may become possible due to a matter-induced shift in the mass of the D mesons[140, 141, 142, 143, 144, 145].

The main difference between the nuclear interactions of slow J/ψ and ψ' is that for the latter there exist subthreshold scattering processes: the charm-exchange process $\psi'+N \rightarrow \Lambda_c + \bar{D}$, the charmonium transition scattering $\psi'+N \rightarrow J/\psi + N$, and generally additional channels where in the latter process instead of a single nucleon excited states are being produced such as $N\pi$, $\Delta\pi$, etc. The processes other than $\psi'+N \rightarrow J/\psi + N$ are beyond the scope of the present discussion. It can only be noted here that due to the discussed relation of the relevant gluonic matrix element to the energy-momentum tensor in QCD, the processes with non-diagonal transitions, such as $N \rightarrow N\pi$, should be suppressed with respect to the diagonal one $N \rightarrow N$. One can also notice that similar transitions from ψ' to lower charmonium states other than J/ψ should also be suppressed in comparison with $\psi' \rightarrow J/\psi$, since those other states cannot be produced in the second order in the leading $E1$ term of the multipole expansion.

The process $\psi' + N \rightarrow J/\psi + N$ for a slow ψ' involves a momentum transfer to the nucleon $q^2 \approx -0.82 \text{ GeV}^2$, so that the previous, essentially static consideration is generally modified by an effect of an unknown form factor $F(q^2)$. For this reason the presented here estimates are somewhat approximate. The cross section for the discussed transitional process is found, using the known value of $\alpha^{(12)}$ in complete analogy with Eq.(115):

$$\sigma(\psi' + N \rightarrow J/\psi + N) \approx 16 \text{ mb} \left(\frac{1 \text{ GeV}}{p_i} \right) |F(q^2)|^2 , \quad (118)$$

where p_i is the c.m. momentum of the initial particles. The inverse-velocity, $1/p_i$, behavior of the cross section is due the subthreshold kinematics of the process. Assuming, conservatively, that the form factor $|F(q^2)|$ suppresses the amplitude by not more than a factor of two, one comes to the conclusion that the cross section of the considered process can reach tens of millibarn at rather moderately low values of the initial momentum p_i .

The discussed process gives rise to a decay rate of the ψ' in a nuclear medium, which can be evaluated[138] as

$$\Gamma(\psi' \rightarrow J/\psi) \approx 70 \text{ MeV} \left(\frac{\rho_N}{0.16 \text{ fm}^{-3}} \right) |F(q^2)|^2 , \quad (119)$$

and is likely reaching tens of MeV at the nominal average nuclear density.

6 Charmonium above the $D\bar{D}$ threshold

The charmonium resonances that are heavier than the $D^0\bar{D}^0$ threshold at 3.73 GeV are kinematically allowed to decay into D meson pairs and are generally expected to be significantly broader than the states below the threshold. The exception from such behavior being for the resonances that might have mass still below the $D^0\bar{D}^{*0}$ threshold at 3.87 GeV and the quantum numbers that forbid their decay into a pair of pseudoscalar mesons, such as those with unnatural spin-parity, $P = (-1)^{J+1}$, or with negative CP parity. Some potential models point at existence in this narrow mass range of such ‘exceptional’ resonances 1D_2 and 3D_2 both having unnatural parity. However no such states have been observed thus far. Instead, an already plentiful and growing suite of very interesting states is being observed at the $D^0\bar{D}^{*0}$ threshold and above, some of which almost definitely cannot be explained as simple $c\bar{c}$ states,

but rather should additionally contain light quarks and/or gluons as dynamical constituents. Thus in this mass range the dynamics of the heavy $c\bar{c}$ pair closely intermixes with the dynamics of charmed meson pairs and with general nonperturbative dynamics in QCD.

6.1 $\psi(3770)$

6.1.1 General Properties

The resonance $\psi(3770)$, or ψ'' , with the quantum numbers $J^{PC} = 1^{--}$ can and does decay into $D\bar{D}$ meson pairs, which explains its relatively large total decay width[27] $\Gamma[\psi(3770)] = 25.2 \pm 1.8$ MeV. The e^+e^- decay width of the $\psi(3770)$, $\Gamma_{ee}[\psi(3770)] = 0.247^{+0.028}_{-0.025}$ keV is about ten times smaller than that of the nearby ψ' . Thus this resonance is considered to be dominantly a 3D_1 state of charmonium with a small admixture of 3S_1 . The latter admixture enhances the e^+e^- decay rate, which otherwise would be very small for a pure 3D_1 state. The amount of mixing can be estimated from the value of Γ_{ee} . Using a simple model with a two-state $\psi' - \psi(3770)$ mixing Rosner[62, 63] estimates the mixing angle as $(12 \pm 2)^\circ$, which certainly agrees with the notion of the ${}^3D_1 - {}^3S_1$ mixing being an $O(v^2/c^2)$ effect and the estimate $v^2/c^2 \approx 0.2$. It should be understood however that the particular estimated value of the mixing provides only an approximate guidance, not only due to its theoretical model dependence, but also because the experimental data, especially for $\psi(3770)$, are still somewhat volatile.

In particular, the data are still not conclusive on the decay properties of $\psi(3770)$, most notably on the fraction of the decay rate that is not associated with the decay $\psi(3770) \rightarrow D\bar{D}$. Namely, the reported by CLEO[146] result for the total resonance production cross section at the maximum of the $\psi(3770)$ peak (at $E_{c.m.} = 3773$ MeV), $\sigma[e^+e^- \rightarrow \psi(3770)] = (6.38 \pm 0.08^{+0.41}_{-0.30})$ nb leaves very little if any room for non- $D\bar{D}$ decays, if combined with their latest measurement[147] of the $D\bar{D}$ production at the same maximum: $\sigma(e^+e^- \rightarrow D\bar{D}) = (6.57 \pm 0.04 \pm 0.10)$ nb. On the other hand, the BES measurement of the total cross section[148] gives $\sigma[e^+e^- \rightarrow \psi(3770)] = (7.25 \pm 0.27 \pm 0.34)$ nb, and their directly reported result[149] for the branching fractions: $\mathcal{B}[\psi(3770) \rightarrow D^0\bar{D}^0] = (46.7 \pm 4.7 \pm 2.3)\%$, $\mathcal{B}[\psi(3770) \rightarrow D^+D^-] = (36.9 \pm 3.7 \pm 4.2)\%$ and $\mathcal{B}[\psi(3770) \rightarrow \text{non-}D\bar{D}] = (16.4 \pm 7.3 \pm 4.2)\%$ leaves an ample room for non- $D\bar{D}$ decays of $\psi(3770)$ and their branching fraction for $\psi(3770) \rightarrow D\bar{D}$ is also in agreement with the CLEO result for $\sigma(e^+e^- \rightarrow D\bar{D})$.

A sizable fraction of non- $D\bar{D}$ decays of $\psi(3770)$ would present a serious difficulty for considering it as a pure $c\bar{c}$ state. Indeed, its total annihilation rate should be small in comparison with such rate for the ψ' , similarly to its leptonic width Γ_{ee} . The radiative and hadronic transitions to lower charmonium levels are expected to be small and in fact are measured to be small: the hadronic transitions to J/ψ all together contribute less than approximately 0.5 % of the total decay rate[150, 151], while the total fraction due radiative decays to $\gamma + \chi_{cJ}$ [152] likely amounts to at most about 1 %. Thus if the non- $D\bar{D}$ decay rate of the $\psi(3770)$ is measured to exceed the small rate that can be accounted for, it would imply an enhanced decay of this resonance into light hadrons. Such enhancement, if found, can be attributed[153, 154] to a presence in the wave function of $\psi(3770)$ of a certain four-quark component: $c\bar{c}u\bar{u}$ and $c\bar{c}d\bar{d}$, where the annihilation of the heavy quark pair is enhanced[155]. A mixture of the $\psi(3770)$ with four-quark states can be viewed as a ‘re-annihilation’[153] of $D\bar{D}$ meson pairs, which are strongly coupled to the resonance. Furthermore, within such mechanism one can expect that in the four-quark component an enhanced violation of the isotopic spin due to the fact that the mass difference between the D^+D^- and $D^0\bar{D}^0$ thresholds, $\Delta \approx 9.6$ MeV, is not much smaller than the excitation energy of the $\psi(3770)$ resonance above these thresholds.

A presence of light quark-antiquark pairs in the wave function of $\psi(3770)$ should generally enhance[154] both the $\pi\pi$ and η transitions to J/ψ with a larger enhancement for the latter transition, which is otherwise suppressed by the flavor SU(3) symmetry. Experimentally[150] the branching fractions are $\mathcal{B}[\psi(3770) \rightarrow \pi^+\pi^- J/\psi] = (0.189 \pm 0.020 \pm 0.020)\%$ and $\mathcal{B}[\psi(3770) \rightarrow \eta J/\psi] = (0.087 \pm 0.033 \pm 0.022)\%$,

so that the ratio of the η emission rate to that of the $\pi^+\pi^-$ pairs is approximately 0.5. Such ratio indicates a significant relative enhancement of the η transition, if compared with the similar ratio, 0.1, for the ψ' decays and given the fact that an increased phase space favors the two-pion transitions more than the η emission.

Furthermore, an isospin violation in the four-quark admixture in $\psi(3770)$ implies a presence of an isovector, $I = 1$, component in its wave function. Such component should then enhance the single-pion transition $\psi(3770) \rightarrow \pi^0 J/\psi$, and, through the $\psi' - \psi(3770)$ mixing, also provide an additional contribution to the amplitude of the decay $\psi' \rightarrow \pi^0 J/\psi$, much needed given the previously discussed mismatch between the data and the theory. Starting with the needed contribution to the latter decay one can estimate[154] the expected rate of the former transition as $\mathcal{B}[\psi(3770) \rightarrow \pi^0 J/\psi] \sim 2 \times 10^{-4}$, which can be compared with the current experimental limit[150] $\mathcal{B}[\psi(3770) \rightarrow \pi^0 J/\psi] < 2.8 \times 10^{-4}$ at 90% C.L.

6.1.2 Isospin Breaking in Production of $D\bar{D}$ pairs at $\psi(3770)$

The production of the $D\bar{D}$ meson pairs at and near the $\psi(3770)$ resonance in e^+e^- annihilation is essentially completely dominated by the electromagnetic current of the charmed quarks, which is a pure isoscalar. However the yield of the two isotopic components in the final state, D^+D^- and $D^0\bar{D}^0$, is not the same. Rather the ratio is measured[147, 149] to be $\sigma(e^+e^- \rightarrow D^+D^-)/\sigma(e^+e^- \rightarrow D^0\bar{D}^0) = 0.79 \pm 0.01 \pm 0.01$ at the maximum of the $\psi(3770)$ peak. This is certainly not unexpected since, as previously mentioned, the mass difference between the charged and neutral D mesons is substantial at the energy of the resonance, and also the Coulomb interaction between the produced slow charged D mesons modifies their production cross section. Usually the effect of the mass difference in the cross section is estimated by the P wave kinematical factor p^3 with p being the momentum of each meson in the c.m. frame. The Coulomb effect in the limit of slow point-like particles produced by a point source reduces to the well known factor $[1 + \pi\alpha/(2v)]$, where v is the c.m. velocity of either of the charged mesons ($v \approx 0.13$ for the D^+D^- pairs produced at the $\psi(3770)$ peak). A straightforward estimate of the product of the kinematical and the Coulomb factors gives $(p_+/p_0)^3 [1 + \pi\alpha/(2v)] \approx 0.75$ in a reasonable agreement with the experimental number for the charged-to-neutral yield ratio. However, it would be premature to conclude that the issue of this ratio is solved. Indeed, a similar estimate does not work at all for the B meson pair production at a similar near-threshold resonance $\Upsilon(4S)$, where the isotopic mass difference for the B mesons is practically nonexistent, and the Coulomb factor amounts to about 1.19, while the most precise data give the yield ratio very close to one[156]: $1.006 \pm 0.036 \pm 0.031$. On the theoretical side, it is well understood[157, 158, 159] that the form factors in the production vertex and in the Coulomb interaction between the charged mesons generally modify the charged-to-neutral yield ratio. Another related effect[160, 161], that modifies both the kinematical and the Coulomb correction factors is the strong interaction between the mesons, which certainly is relevant since there is a resonance in the production channel.

Due to small velocity of the mesons near the resonance energy one can apply the methods of the standard nonrelativistic quantum mechanics. With these methods the strong interaction is assumed to be confined to a certain radius $r < a$, and that in the isotopically symmetric case the (P wave) states of meson pairs with definite isospin, $I = 0$ and $I = 1$, are characterized at $r > a$ by the scattering phases δ_0 and δ_1 . Both phases behave as p^3 near the threshold, while the $I = 0$ phase δ_0 also makes a rapid variation across the isoscalar $\psi(3770)$ resonance. The isospin violating effects due to the mass difference and the Coulomb interaction (including the form factor) can be treated as being due to a difference $\delta V(r)$ at distances $r > a$ in the potential for the D^+D^- and $D^0\bar{D}^0$ channels. Then if a source producing the meson pairs is localized entirely within the region of strong interaction, and the production amplitude has the isotopic composition $A = A_0|I = 0\rangle + A_1|I = 1\rangle$, the charge-to-neutral

yield ratio $R^{c/n}$ is given to the first order in δV by the formula[161]

$$R^{c/n} = \left| \frac{A_0 + A_1}{A_0 - A_1} \right|^2 \left\{ 1 + \frac{1}{v} \text{Im} \left[\frac{A_0 e^{2i\delta_1} - A_1 e^{2i\delta_0}}{A_0 - A_1} \int_a^\infty e^{2ipr} \left(1 + \frac{i}{pr} \right)^2 \delta V(r) dr \right] \right\}. \quad (120)$$

In the case of an essentially pure isoscalar source, relevant to the charmed meson pair production in e^+e^- annihilation, this general expression reduces to the following

$$R^{c/n} = 1 + \frac{1}{v} \text{Im} \left[e^{2i\delta_1} \int_a^\infty e^{2ipr} \left(1 + \frac{i}{pr} \right)^2 \delta V(r) dr \right], \quad (121)$$

so that the strong interaction effect in the discussed corrections is determined by the scattering phase in the isovector state δ_1 and does not depend on the resonant isoscalar phase δ_0 .

The dependence of the effect on the parameter a is an inevitable consequence of a P wave dynamics. Only in the limit of vanishing phase δ_1 this parameter can be set equal to zero. It can be also mentioned that if one considers the Coulomb interaction of the charged D mesons as that of point particles, the effect of this interaction as well as that of the mass difference corresponds to the potential difference $\delta V = \Delta - \alpha/r$, the result of the simplified approach is recovered after integration in Eq.(121) down to $a = 0$: $R^{c/n} = 1 - 3\Delta/(2vp) + \pi\alpha/(2v)$. For a nonvanishing δ_1 the correction depends in an essential way on both a and δ_1 [161]. Due to the p^3 dependence of this scattering phase one can expect a measurable variation of the ratio $R^{c/n}$ with energy near the threshold. An experimental study of this variation can thus provide an information on the strong interaction between heavy mesons, which information would not be available by other means.

6.2 X(3872)

6.2.1 General Properties

In the summer of 2003 the Belle Collaboration announced[162] an observation of a narrow resonance $X(3872)$ produced in the decays $B \rightarrow K X$ and decaying as $X(3872) \rightarrow \pi^+\pi^- J/\psi$. The statistical significance of the new peak in the invariant mass of $\pi^+\pi^- J/\psi$ was in excess of 10σ . Shortly after the initial discovery the new resonance was confirmed by observation with similar significance in the inclusive production in $p\bar{p}$ collisions at the Tevatron[163, 164] and by another independent observation in the B decays[165]. The first observed peculiar features of this resonance were its small width, $\Gamma_X < 2.3$ MeV[162] and the exceptional proximity of its mass to the threshold of $D^0\bar{D}^{*0}$: with the recent improvement in the precision of the D^0 mass[166], which placed the $D^0\bar{D}^{*0}$ threshold at 3871.81 ± 0.36 MeV, the mass of the $X(3872)$ corresponds to $M_X - M(D^0\bar{D}^{*0}) = -0.6 \pm 0.6$ MeV.

The small width of $X(3872)$ implies that its decay into $D\bar{D}$ is forbidden either by its unnatural spin-parity, or by negative CP parity, which, as discussed, would be possible for certain states of charmonium. However such states would undergo radiative transitions into $\gamma + \chi_{cJ}$, which transitions were not observed in the experiment[162]. An analysis[168] of the angular correlations[167] in the process with the decay $X \rightarrow \pi^+\pi^- J/\psi$ prefers the assignment $J^{PC} = 1^{++}$. A similar analysis by CDF[169] allows either 1^{++} or 2^{-+} . The latter assignment however would greatly suppress the decay of X to $D^0\bar{D}^0\pi^0$, which is very close to its threshold. It is generally believed that the observation of this decay mode by Belle [170] and BaBar[171] rules out the possibility of $J^{PC} = 2^{-+}$.

The positive C parity of $X(3872)$ is in fact directly mandated by the observations[172, 173] of the decay $X \rightarrow \gamma J/\psi$. When combined with the existence of the discovery mode $X \rightarrow \pi^+\pi^- J/\psi$ this implies that the pions in the latter decay have to be in a C-odd state, and thus the total isospin of the pion pair has to be equal to one! In particular no emission of a $\pi^0\pi^0$ pion pair can take place. Thus the X resonance definitely cannot be a pure $c\bar{c}$ system, but has to contain light quarks in its wave function.

Furthermore, the Belle data[172] also indicate that the decay $X(3872) \rightarrow \pi^+\pi^-\pi^0 J/\psi$ has a rate approximately equal to that of the decay into $\pi^+\pi^- J/\psi$. By the G parity a system of three pions in a state with a fixed C parity cannot have the same isospin as a system of two pions. Specifically, at the negative C parity the only possible values of the isospin are $I(2\pi) = 1$, and $I(3\pi) = 0$, or 2. Therefore, not only the isospin of X is nontrivial, but it is not definite altogether. This is also supported by the negative results of the search[174] for charged states, which would be the isospin partners of X if it had $I = 1$ and the isospin was a good quantum number.

6.2.2 $X(3872)$ as a (Dominantly) Molecular State

The unusual properties of the $X(3872)$ state gave rise to the suggestion[175, 176, 177, 178] that the wave function of this state has a significant ‘molecular’ component made out of mesons rather than out of quarks. In particular the quantum numbers $J^{PC} = 1^{++}$ imply this component in X contains the state $D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$ in the S wave, which is quite natural, given the extreme proximity of the mass of X to the corresponding threshold. Furthermore, the threshold for the pairs of charged mesons, $D^+D^{*-} + D^-D^{*+}$ is heavier by $\Delta \approx 8$ MeV, and this mass gap is large in the scale of the possible binding energy w for the neutral mesons. For this reason the isospin is badly broken in $X(3872)$ and the wave functions of the $D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$ and $D^+D^{*-} + D^-D^{*+}$ components are significantly different, which then explains the unusual isotopic decay properties of the X resonance.

An existence of molecular states of loosely bound heavy hadrons was argued on general grounds long ago[155] and a molecular interpretation was considered[179] for explaining the properties of the then already known $\psi(4040)$ resonance. The argument for the existence of the bound states is essentially quite straightforward: the strong force between hadrons containing heavy and light quarks arising due to the interaction between the light components does not depend on the mass of the heavy quark. Therefore at a sufficiently large heavy quark mass there inevitably are bound states in the channels where the strong interaction gives an attraction. However it had to be tested experimentally, whether the charmed quark is ‘heavy enough’ to form such states in some channels.

It should be understood however that there is no reason to expect that only the molecular component is present in the wave function and that it determines all of the properties of the $X(3872)$ boson. Rather one should consider the wave function in terms of a general Fock decomposition:

$$\psi_X = a_0 \psi_0 + \sum_i a_i \psi_i , \quad (122)$$

where ψ_0 is the state of the neutral D mesons $(D^0\bar{D}^{*0} + \bar{D}^0D^{*0})/\sqrt{2}$, while ψ_i refer to ‘other’ hadronic states. Due to the extreme proximity of the mass of X to the $D^0\bar{D}^{*0}$ threshold, the ψ_0 part should be dominant at long distances. Indeed, assuming that the mass of X is below the threshold by the binding energy w : $m_{D^0} + m_{D^{*0}} - M_X = w$, the spatial extent of the ψ_0 is determined as $(m_D w)^{-1/2} \approx 5$ fm (1 MeV/ w) $^{1/2}$, and ψ_0 thus describes the ‘peripheral’ part of the wave function, in fact beyond the range of strong interaction. On the other hand, the ‘other’ states in the sum in the Fock decomposition (122) are localized at shorter distances and constitute the ‘core’ of the $X(3872)$ wave function. In other terms, one may think of this picture as that of a mixing in $X(3872)$ of the molecular component $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ with ‘other’ states, such as e.g. a ‘pure’ $c\bar{c}$ charmonium, which then has to be in a 3P_1 state, also favored by the heavy quark spin selection rule[180]. The notion of X being a ‘molecular’ system is helpful inasmuch as the probability weight $|a_0|^2$ of the meson component ψ_0 makes a large portion of the total normalization. In particular the model of Ref.[181] includes S and D wave states of the neutral and charged charmed meson pairs as well as the channels $\rho J/\psi$ and $\omega J/\psi$, and estimates the weight factor of the S wave $(D^0\bar{D}^{*0} + \bar{D}^0D^{*0})$ component as 70-80% at $w \approx 1$ MeV.

Since the internal composition of the $X(3872)$ can be quite different at different distances, one or the other part of the Fock decomposition (122) may be important in specific processes. It appears that

the pionic transitions from $X(3872)$ to J/ψ are determined by a long distance dynamics, where the $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component dominates, so that the isospin states are mixed, and the $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ transitions have approximately the same strength. The production of X however is determined by short distances, and proceeds through the core component, which is approximately an isospin singlet, as evidenced[182] by a comparable relative rate of the decays $B^+ \rightarrow X K^+$ and $B^0 \rightarrow X K^0$, while an exclusive contribution of the molecular $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ state would likely correspond to a strong suppression[183, 184] of the $B^0 \rightarrow X K^0$ decay in comparison with $B^+ \rightarrow X K^+$.

The mesons in the $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component move freely beyond the range of the strong interaction, where their wave function in the coordinate space is given by

$$\phi_n(r) = c \frac{\exp(-\kappa_n r)}{r}, \quad (123)$$

where κ_n is determined by the binding energy w and the reduced mass $m_r \approx 966$ MeV in the $D^0\bar{D}^{*0}$ system as $\kappa_n = \sqrt{2m_r w}$. The normalization coefficient c determines the statistical weight of the $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component in $X(3872)$, and its definition is correlated with that of the coefficient a_0 in the Fock decomposition (122). We resolve this ambiguity in the definition by requiring that the coordinate wave function of the neutral meson pair be normalized to one, so that the statistical weight of the state $(D^0\bar{D}^{*0} + D^{*0}\bar{D}^0)/\sqrt{2}$ is given as $|a_0|^2$. If the wave function of the form (123) is used down to $r = 0$, this requirement corresponds to $c = \sqrt{\kappa_n/(2\pi)}$.

The dominance of the $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ at long distances translates into a substantial isospin violation in the processes determined by the ‘peripheral’ dynamics, examples of which are apparently the observed decays $X \rightarrow \pi^+\pi^-J/\psi$ and $X \rightarrow \pi^+\pi^-\pi^0J/\psi$. It is quite likely however that this isospin-breaking behavior is only a result of the ‘accidentally’ large mass difference $\Delta \approx 8$ MeV between D^+D^{*-} and $D^{*0}\bar{D}^{*0}$. Therefore it is natural to expect that at shorter distances within the range of the strong interaction the isospin symmetry is restored and at those distances the wave function of $X(3872)$ is dominated by $I = 0$. In this picture the wave function of a $D^+D^{*-} + D^-D^{*+}$ state within the region beyond the range of the strong interaction can be found from Eq.(123) by requiring that at short distances the pairs of charged and neutral mesons combine into an $I = 0$ state⁷, so that the wave function of the charged meson pair has the form

$$\phi_c(r) = c \frac{\exp(-\kappa_c r)}{r}, \quad (124)$$

where $\kappa_c = \sqrt{2m_r(\Delta + w)} \approx 125$ MeV. It should be noticed that both the neutral (Eq.(123)) and the charged (Eq.(124)) meson wave function have the same normalization factor c (determined by κ_n) and differ only in the exponential power. This simple picture allows one to estimate the relative statistical weight of the charged and neutral D meson components in the $X(3872)$:

$$\lambda \equiv \frac{|\langle X | D^+D^{*-} + D^-D^{*+} \rangle|^2}{|\langle X | D^0\bar{D}^{*0} + D^{*0}\bar{D}^0 \rangle|^2} = \frac{\kappa_n}{\kappa_c}. \quad (125)$$

Clearly, the wave functions in Eq.(123) and Eq.(124) cannot be applied at short distances in the region of strong interaction, where the mesons overlap with each other and cannot be considered as individual particles. In order to take into account this behavior an ‘ultraviolet’ cutoff should be introduced. One widely used method for introducing such cutoff is to consider the meson wave functions only down to a finite distance r_0 , at which distance the boundary condition of the state being that with $I = 0$ is imposed. An alternative, somewhat more gradual cutoff, described by parameter Λ , can

⁷The effects of the Coulomb interaction between the charged mesons are neglected here.

be introduced[184] by subtracting from the wave functions (123) and (124) an expression $ce^{-\Lambda r}/r$. It should be noticed, that such regularization also results in a modification of the normalization coefficient c , which for the gradual cutoff takes the form

$$c = \sqrt{\frac{\kappa_n}{2\pi}} \frac{\sqrt{\Lambda(\Lambda + \kappa_n)}}{\Lambda - \kappa_n} . \quad (126)$$

One can also readily see, that an introduction of any such cutoff eliminates relatively more of the charged meson wave function than of the neutral one, thus reducing the estimate of the relative statistical weight as compared to that in Eq.(125), so that Eq.(125) gives in fact the upper bound for the ratio.

6.2.3 Peripheral Decays to $D^0\bar{D}^0\pi^0$ and $D\bar{D}\gamma$

The ‘molecular’ component of the $X(3872)$ dominating at large distances, the periphery, should give rise to decays to $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ [178, 185]. The underlying processes in these decays are the decays of the very weakly bound D^{*0} meson $D^{*0} \rightarrow D^0\pi^0$ and $D^{*0} \rightarrow D^0\gamma$, and the charge-conjugate decays of the \bar{D}^{*0} . These decays are relatively well studied for the D^* mesons, and the relevant rates can be deduced from the data in the Tables[27] as

$$\Gamma_\pi \equiv \Gamma(D^{*0} \rightarrow D^0\pi^0) = 43 \pm 10 \text{ keV} \quad \text{and} \quad \Gamma_\gamma \equiv \Gamma(D^{*0} \rightarrow D^0\gamma) = 26 \pm 6 \text{ keV} . \quad (127)$$

There is an interference between the amplitude of the decay of D^{*0} and \bar{D}^{*0} in state with a fixed C parity of the initial meson pair. For the C-even X resonance the sign of the interference is positive for the decay into $D^0\bar{D}^0\pi^0$ and is negative for the decay into $D^0\bar{D}^0\gamma$. The differential over the Dalitz plot rate of the decay $X(3872) \rightarrow D^0\bar{D}^0\pi^0$ can then be found as[178]

$$d\Gamma(X \rightarrow D^0\bar{D}^0\pi^0) = |a_0|^2 \frac{\Gamma_\pi(\vec{q}_1 + \vec{q}_2)^2}{12\pi^2 p_0^3} |\phi(\vec{q}_1) + \phi(\vec{q}_2)|^2 d\vec{q}_1^2 d\vec{q}_2^2 , \quad (128)$$

where, $p_0 = 43$ MeV is the π^0 momentum in the decay of a free D^{*0} meson, \vec{q}_1 and \vec{q}_2 are the momenta of the D^0 and \bar{D}^0 mesons in the rest frame of X , and $\phi(\vec{q})$ is the momentum-space wave function corresponding to that in Eq.(123):

$$\phi(\vec{q}) = \frac{4\pi c}{\vec{q}^2 + \kappa_n^2} . \quad (129)$$

The significance of the interference and the total rate of the decay depend on the binding energy w :

$$\Gamma(X \rightarrow D^0\bar{D}^0\pi^0) = |a_0|^2 \Gamma_\pi [A(w) + B(w)] , \quad (130)$$

where $A(w)$ describes the incoherent contribution of the decays of individual D^{*0} and \bar{D}^{*0} , and $B(w)$ describes the effect of the interference between these two processes. The result of a numerical calculation of the terms A and B with the wave function from Eq.(129) is shown in Fig.5. It is seen from the plot, that the interference between the two wave functions in Eq.(128) significantly enhances the decay from a C-even state.

The decay $X(3872) \rightarrow D\bar{D}\gamma$ can be considered in a similar way, and the differential decay rate is found as

$$d\Gamma(X \rightarrow D\bar{D}\gamma) = \Gamma_\gamma \frac{a_0^2}{2} \left(\frac{\omega}{\omega_0}\right)^3 \left[\phi\left(\frac{\vec{k}}{2} + \vec{p}\right) - \phi\left(\frac{\vec{k}}{2} - \vec{p}\right) \right]^2 \frac{d^3 p}{(2\pi)^3} , \quad (131)$$

where $\omega_0 = 137$ MeV stands for the photon energy in the corresponding decay $D^{*0} \rightarrow D\gamma$, ω is the energy of the photon, and $\vec{p} = (\vec{p}_D - \vec{p}_{\bar{D}})/2$ is the momentum of the D meson in the c.m. frame of the

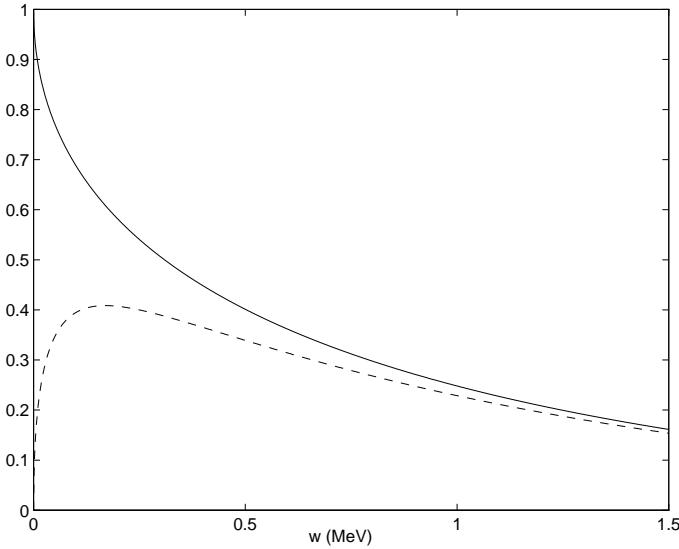


Figure 5: The non-coherent contribution $A(w)$ (solid line) and the interference term $B(w)$ (dashed) as defined in Eq.(130), calculated by a numerical integration in Eq.(128).

final $D\bar{D}$ meson pair. An integration with the wave function with the cutoff Λ at short distances gives the expression for the total decay rate

$$\Gamma(X \rightarrow D^0\bar{D}^0\gamma) = \Gamma_\gamma a_0^2 \left[1 - \frac{2\kappa_n}{\omega_0} \frac{\Lambda(\Lambda + \kappa_n)}{(\Lambda - \kappa_n)^2} \left(\arctan \frac{\omega_0}{2\kappa_n} + \arctan \frac{\omega_0}{2\Lambda} - 2 \arctan \frac{\omega_0}{\Lambda + \kappa_n} \right) \right]. \quad (132)$$

At $\omega_0 = 137$ MeV and $\kappa_n \approx 44$ MeV (corresponding to the binding energy $w = 1$ MeV) the numerical value of the expression in the square braces, the interference factor, varies between 0.36 at $\Lambda \rightarrow \infty$ and 0.61 at $\Lambda = 200$ MeV. The spectrum of the photon energies is shown in Fig.6. As can be expected on general grounds this spectrum peaks near the energy corresponding to the decay $D^{*0} \rightarrow D^0\gamma$ with the spread induced by the slow ‘Fermi motion’ of the initial meson in the loosely bound state.

In the peripheral contribution the decay to $D^+D^-\gamma$ is greatly suppressed in comparison with $D^0\bar{D}^0\gamma$. The suppression results from three contributing factors[185]: the small rate of the decay $D^{*+} \rightarrow D^+\gamma$, the relatively small statistical weight of the pair of charged mesons in $X(3872)$ and the stronger negative interference for the charged mesons located at shorter distances within the X resonance wave function. One can also argue[185] that the appearance of the pairs of charged mesons as a result of rescattering $D^0\bar{D}^0 \rightarrow D^+D^-$ should be only a minor effect.

A quite different final state composition should be expected for the $D\bar{D}\gamma$ final state from the ‘core’ component of the $X(3872)$. In particular this component can give rise to decays through the intermediate $\psi(3770)$ resonance: $X(3872) \rightarrow \psi(3770)\gamma \rightarrow D\bar{D}\gamma$ in which case the photon spectrum should have a peak at $\omega \approx 100$ MeV and the isotopic composition of the final $D\bar{D}$ state should be the same as in the decay of $\psi(3770)$.

6.2.4 $e^+e^- \rightarrow \gamma X(3872)$ as an Alternative Source of $X(3872)$

The resonance $X(3872)$ is observed experimentally only in the decays of B mesons $B \rightarrow X K$ and in inclusive production in proton - antiproton collisions at the Tevatron. Both these types of processes present significant challenges for precision measurements of the parameters of the resonance. In particular, neither the total width of $X(3872)$ is yet resolved (the current limit is $\Gamma_X < 2.3$ MeV), nor its mass is known with a precision sufficient to determine the mass gap w from the D^0D^{*0} threshold. A

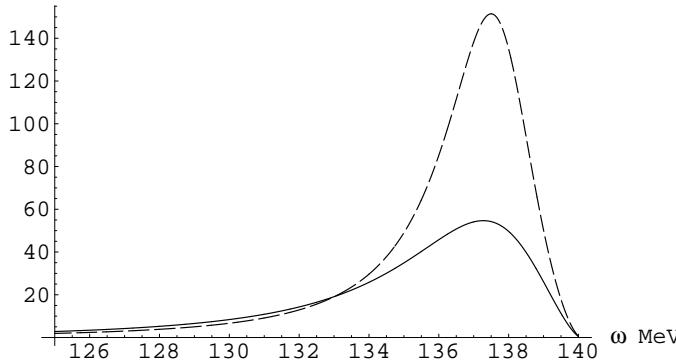


Figure 6: The photon spectrum in the decay $X(3872) \rightarrow D^0 \bar{D}^0 \gamma$ for $\kappa_n = 44$ MeV (solid) and for $\kappa_n = 24$ MeV (dashed), both at $\Lambda \rightarrow \infty$. The vertical scale is in arbitrary units.

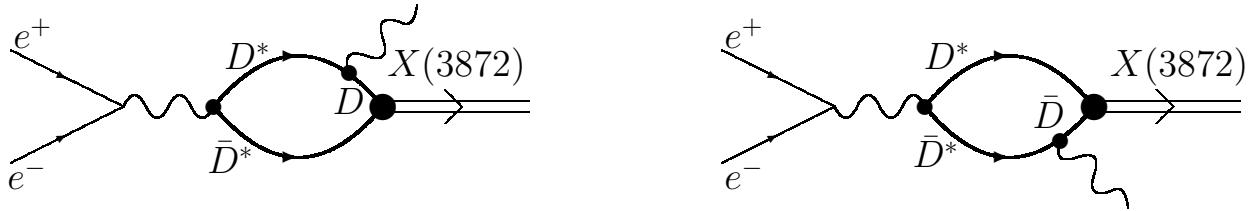


Figure 7: The production process $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \rightarrow \gamma X(3872)$.

viable alternative source of the $X(3872)$ can be provided by the process $e^+e^- \rightarrow \gamma X(3872)$ [186] at the c.m. energy within few MeV of the $D^{*0}\bar{D}^{*0}$ threshold, where the kinematical simplicity of the process would hopefully allow more detailed studies of $X(3872)$.

The cross section of the discussed process can be estimated by using the unitarity relation and considering the process, shown in Fig.7 $e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \rightarrow \gamma X(3872)$, and also using the amplitude of the known decay $D^{*0} \rightarrow \gamma D^0$. The amplitude for production of the $D^*\bar{D}^*$ pair in e^+e^- annihilation at a small energy above the threshold $E = 2M(D^{*0}) + W$ can be generically written in the form

$$A(e^+e^- \rightarrow D^{*0}\bar{D}^{*0}) = A_0 (\vec{j} \cdot \vec{p}) (\vec{a} \cdot \vec{b})^* + \frac{3}{2\sqrt{5}} A_2 j_i p_k \left[a_i b_k + a_k b_i - \frac{2}{3} \delta_{ik} (\vec{a} \cdot \vec{b}) \right]^*, \quad (133)$$

where $\vec{j} = (\bar{e}\vec{\gamma}e)$ stands for the current of the incoming electron and positron, \vec{p} is the momentum of one of the mesons (D^{*0} for definiteness) in the c.m. frame, and A_0 and A_2 are the factors corresponding to production of the vector meson pair in the states with respectively the total spin $S = 0$ and $S = 2$. It can be also noted that the amplitude in Eq.(133) describes the production of mesons in the P wave. Another kinematically possible amplitude, the F -wave, should be small near the threshold, i.e. at a small W . Both A_0 and A_2 are generally functions of the excitation energy W . Furthermore, their dependence on the energy near the threshold is known to be nontrivial due to the $\psi(4040)$ resonance[27], with possible further complications in the immediate vicinity of the threshold[187, 188]. Neither the relative magnitude nor the relative phase of the amplitudes A_0 and A_2 is presently known, but both of these can be measured from angular correlations[189]. These amplitudes determine the total cross

section for production of $D^{*0}\bar{D}^{*0}$ in e^+e^- annihilation:

$$\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0}) = \int |A(e^+e^- \rightarrow D^{*0}\bar{D}^{*0})|^2 2\pi \delta \left(W - \frac{p^2}{m} \right) \frac{d^3 p}{(2\pi)^3} = C \frac{m p^3}{2\pi} \left(|A_0|^2 + |A_2|^2 \right), \quad (134)$$

where $m = M(D^{*0})$, $p = |\vec{p}|$, and C is an overall constant related to the average value of the current $|\vec{j}|^2$. The specific value of the latter constant is not essential for the discussed estimate, since it cancels in the ratio of the cross sections. The latter ratio thus can be expressed in terms of the amplitudes A_0 and A_2 and of the parameters of the decay $D^{*0} \rightarrow \gamma D^0$, the rate Γ_γ and the photon energy ω_0 ,

$$\frac{\sigma_{\text{Abs}}(e^+e^- \rightarrow D^{*0}\bar{D}^{*0} \rightarrow \gamma X)}{\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0})} = |a_0|^2 \frac{\Gamma_\gamma m \omega \kappa_n}{2\omega_0^3 p} F^2 \frac{|A_0 - A_2/\sqrt{5}|^2 + (9/20)|A_2|^2}{|A_0|^2 + |A_2|^2}, \quad (135)$$

where ω is the energy of the photon, and the form factor F is defined through the wave function $\phi(\vec{q})$ (Eq.(129) as

$$F = \frac{1}{2} \int_{-1}^1 (\vec{p} \cdot \vec{k}) \phi \left(\vec{p} - \frac{\vec{k}}{2} \right) d \cos \theta, \quad (136)$$

with θ being the angle between the D^* c.m. momentum \vec{p} and the photon momentum \vec{k} . The explicit form of the form factor is

$$F = \frac{c}{p \omega} \left[\left(p^2 + \frac{\omega^2}{4} + \kappa_n^2 \right) \ln \frac{(p + \omega/2)^2 + \kappa_n^2}{(p - \omega/2)^2 + \kappa_n^2} - \left(p^2 + \frac{\omega^2}{4} + \Lambda^2 \right) \ln \frac{(p + \omega/2)^2 + \Lambda^2}{(p - \omega/2)^2 + \Lambda^2} \right] \quad (137)$$

with the normalization coefficient c given by Eq.(126).

The ‘absorptive’ cross section σ_{Abs} in Eq.(135) is (most likely) not the actual value of the cross section, since the amplitude of the process $e^+e^- \rightarrow \gamma X$ can receive contribution from other mechanisms. Nevertheless it is instructive to examine the numerical value and the behavior with energy of this quantity as given by Eq.(135). The dependence on the c.m. energy of the factor $(\kappa_n/p) F^2$ is shown in Fig.8 for two representative values of the ‘molecular’ binding energy in $X(3872)$, $w = 1$ MeV ($\kappa_n \approx 44$ MeV) and $w = 0.3$ MeV ($\kappa_n \approx 24$ MeV). This factor peaks at the energy where $p \approx \omega/2 \approx 70$ MeV. The appearance of such peak is easily understood qualitatively: at $\vec{p} \approx \vec{k}/2$ the D^0 meson emerging from the emission of the photon in $D^{*0} \rightarrow D^0\gamma$ moves slowly relative to the \bar{D}^{*0} and forms a loosely bound state.⁸ The width of the peak is clearly determined by the parameter κ_n .

As is seen from the plots of Fig.8 the numerical value of the factor $(\kappa_n/p) F^2$ near its peak is of order one. Another factor in Eq.(135), $\Gamma_0 m \omega / (2\omega_0^3) \approx \Gamma_0 m / (2\omega_0^2) \approx 1.5 \times 10^{-3}$, sets the overall scale of the discussed cross section. The factor in Eq.(135) depending on the presently unknown ratio of the (generally complex) amplitudes A_0/A_2 , takes values between 0.34 (at $A_0/A_2 \approx 0.68$) and 1.31 (at $A_0/A_2 \approx 1.47$), and can thus be considered as being of order one. Finally, the statistical weight factor $|a_0|^2$, as discussed, is likely to be a large fraction of one. Summarizing these numerical estimates, the value of the ratio in Eq.(135) at the peak can be estimated as being of order 10^{-3} , although the uncertainty is presently large.

In absolute terms, the measured[187, 188] cross section $\sigma(e^+e^- \rightarrow D^{*0}\bar{D}^{*0})$ at $E = 4015$ MeV, i.e. at the energy above the $D^{*0}\bar{D}^{*0}$ threshold $W \approx 1.6$ MeV is about 0.15 nb. This cross section grows from the threshold as p^3 . With this factor taken into account the peak of the quantity $\sigma_{\text{Abs}}(e^+e^- \rightarrow \gamma X)$ shifts to a slightly higher value of p , $p \approx 100$ MeV, corresponding to $W \approx 5$ MeV, where it should be numerically of the order of 1 pb.

The considered mechanism of the process $e^+e^- \rightarrow \gamma X$ describes a ‘soft’ production of its peripheral $D^0\bar{D}^{*0} + D^{*0}\bar{D}^0$ component in radiative transitions from slow $D^{*0}\bar{D}^{*0}$ pairs. Other intermediate states

⁸The same situation arises at $\vec{p} \approx -\vec{k}/2$ for the \bar{D}^0 meson emerging from $\bar{D}^{*0} \rightarrow \bar{D}^0\gamma$.

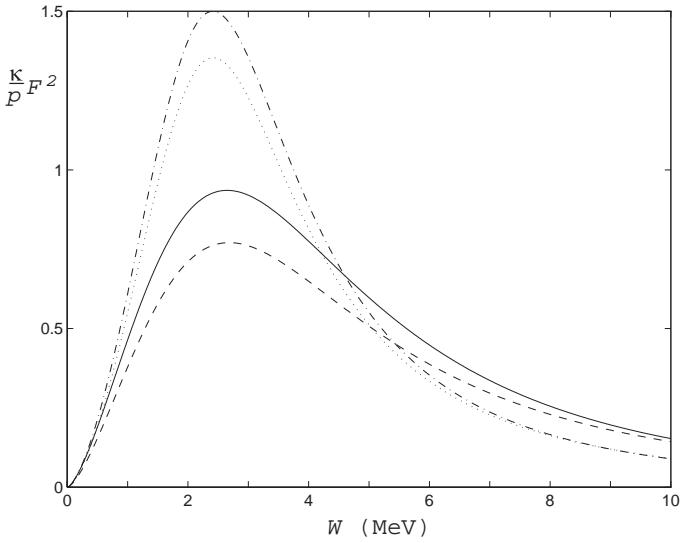


Figure 8: The factor $\kappa_n F^2/p$ vs. the excitation energy W above the $D^{*0}\bar{D}^{*0}$ threshold at representative values of the binding energy w in $X(3872)$ and the ultraviolet cutoff parameter Λ : $w = 1$ MeV, $\Lambda = 200$ MeV (solid), $w = 1$ MeV, $\Lambda = 300$ MeV (dashed), $w = 0.3$ MeV, $\Lambda = 200$ MeV (dashdot), and $w = 0.3$ MeV, $\Lambda = 300$ MeV (dotted).

with charmed meson pairs, i.e. $D^+D^{*-} + D^-D^{*+}$, $D\bar{D}$ and $D\bar{D}^*$ ($\bar{D}D^*$), can potentially contribute to the discussed process $e^+e^- \rightarrow \gamma X$. However, one can readily see that in either of these processes the charmed meson emerging after the emission of the photon is very far off the mass shell in the scale of κ_n . Thus none of such processes can proceed due to the long-distance peripheral component of the $X(3872)$ resonance, but rather is determined by the short-distance dynamics of the ‘core’ of X . For this reason such contributions, as well as other possible mechanisms related to the ‘core’ dynamics, should be smooth functions of the c.m. energy on the scale of few MeV around the $D^{*0}\bar{D}^{*0}$ threshold, where the considered absorptive amplitude experiences a significant variation. Therefore even under the most conservative (and quite unlikely) assumption that these mechanisms cancel the contribution of the latter amplitude near its maximum, such cancellation cannot take place at all energies in the considered range. Thus the cross section of the process $e^+e^- \rightarrow \gamma X$ at an energy within few MeV of the $D^{*0}\bar{D}^{*0}$ threshold has to be at least as large as the estimates for σ_{Abs} near its maximum i.e. of the order of 1 pb. The latter is a conservative estimate, since one cannot exclude that the contribution of those ‘other’ mechanisms exceeds the calculated amplitude and that the actual cross section is larger than σ_{Abs} .

6.2.5 One- and Two-Pion Transitions from $X(3872)$ to χ_{cJ}

Although the bulk of the data on $X(3872)$ indicate that it is very likely to be related to dynamics of $D^0\bar{D}^{*0}$ charmed meson pairs, a possibility is still being considered[190, 191] that the observed properties of $X(3872)$ can be, to an extent, mimicked by a 2^3P_1 state of charmonium, so that any ‘molecular’ admixture would be viewed as a secondary effect due to the coupling to the $D\bar{D}^*$ states. In this picture the main available indicator of a significant isospin violation in $X(3872)$, the approximately equal rate of the decays $X(3872) \rightarrow \rho J/\psi \rightarrow \pi^+\pi^-J/\psi$ and $X(3872) \rightarrow \omega J/\psi \rightarrow \pi^+\pi^-\pi^0J/\psi$ is explained by the kinematical suppression of the isospin-allowed transition $X(3872) \rightarrow \omega J/\psi$.

The transitions from $X(3872)$ to the χ_{cJ} charmonium states with emission of one or two pions, which can be studied in addition to the observed processes $X(3872) \rightarrow \pi^+\pi^-J/\psi$ and $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi$,

may prove to be instrumental in further exploration of the X resonance. Such transitions may be accessible for experimental observation and may hold the clue to understanding the isotopic structure of the $X(3872)$ and of the prominence of the four-quark component in its internal dynamics. The characteristics of such transitions are generally completely different between the possible charmonium and molecular components of the resonance $X(3872)$. The rate of the one-pion transition relative to the process with two pions is sensitive to the $I = 1$ four-quark component of the $X(3872)$, while the isoscalar four-quark component should give rise to relative rates of two-pion transitions to the χ_{cJ} states with different J , which are very likely at variance from those expected for transitions between charmonium levels[192].

The pion transitions from a charmonium excited 3P_1 state are described by the multipole expansion. The two-pion emission is not suppressed by the isotopic symmetry and dominantly proceeds to the χ_{c1} state. Indeed, this is the only final state in the transitions from 3P_1 where the pions can be emitted in the S wave, for which the pion pair creation by the gluonic fields is enhanced by the conformal anomaly in QCD (Eq.(67)). A transition to the χ_{c2} resonance requires a presence of a D wave, proportional to a small parameter $\kappa \approx 0.2$, and is also suppressed by smaller available phase space. Numerically, the suppression turns out to be quite strong, about a factor of 10^{-4} [192]. Moreover, due to the spin-parity properties the χ_{c0} resonance cannot be produced in either S or D wave dipion emission, so that such transition arises only starting with the fourth power of pion momenta in the chiral expansion and should also be quite small. Furthermore, if the quarkonium matrix element (the chromo-polarizability) for the ${}^3P_1 \rightarrow {}^1P_1$ transition is evaluated from the known amplitude of the transition $\psi' \rightarrow \pi\pi J/\psi$, the transition rate can be estimated[192] as $\Gamma({}^3P_1 \rightarrow \pi^+\pi^- {}^1P_1) \approx 1.5 \text{ keV}$, which is most likely only a tiny fraction of the total width of $X(3872)$.

The isospin-violating single pion transitions in charmonium, ${}^3P_1 \rightarrow \pi^0 {}^1P_J$ can also be described within the multipole expansion, similarly to $\psi' \rightarrow \pi^0 J/\psi$. The ratio of the transition rates Γ_J to final states with different J is then given by

$$\Gamma_2 : \Gamma_1 : \Gamma_0 = 3p_{\pi(2)}^3 : 5p_{\pi(1)}^3 : 0 \approx 1 : 2.70 : 0, \quad (138)$$

where $p_{\pi(J)}$ stands for the pion momentum in the corresponding process. One can notice that the χ_{c0} final state is not accessible in this transition too. It can be noted however that in the case of pure charmonium the single pion transition should be even weaker than the two-pion: in the ratio of the rates the unknown quarkonium matrix element cancels, and one finds[192]

$$\frac{\Gamma({}^3P_1 \rightarrow \chi_{c1}\pi^0)}{\Gamma({}^3P_1 \rightarrow \chi_{c1}\pi^+\pi^-)} \approx 0.04. \quad (139)$$

A somewhat different pattern of the transition rates can be expected if the resonance $X(3872)$ is dominantly a molecular state or, generically, is a four-quark state. Then the pion transitions to the charmonium states χ_{cJ} can be treated as a ‘shake off’ of the light quarks. In particular, the spin dependent ‘heavy-light’ quark interaction is proportional to the inverse power of the heavy quark mass m_Q^{-1} , so that any exchange of the polarization between the light and heavy degrees of freedom is expected to be suppressed. Neglecting such exchange one can arrive at the following estimate of the relative rate of the single-pion transitions:

$$\Gamma_0 : \Gamma_1 : \Gamma_2 = 4p_{\pi(0)}^3 : 3p_{\pi(1)}^3 : 5p_{\pi(2)}^3 \approx 2.88 : 0.97 : 1, \quad (140)$$

so that unlike for a pure charmonium the rate of the transition to the lowest χ_{cJ} state should be the largest. Furthermore, due to the apparent strong isospin violation in the wave function of the $X(3872)$ resonance, the single pion process should not be suppressed by the isospin, and in fact is likely to dominate over the two-pion transitions. For the relative strength of the latter decays to χ_{cJ} with different J , it can be mentioned that the transition to χ_{c0} is still suppressed in the chiral expansion by the spin-parity properties, while the χ_{c2}/χ_{c1} ratio is uncertain with the kinematics obviously favoring the χ_{c1} final state.

6.2.6 $X(3872)$ as a Virtual State

The expected decay $X(3872) \rightarrow D^0\bar{D}^0\pi^0$ was sought for and observed[170, 171], however the characteristics of the observed process do not quite look like what one would expect for decay of a bound state. Namely, the experimental study of the B meson decays $B \rightarrow D^0\bar{D}^0\pi^0K$ [170, 171] and $B \rightarrow D^0\bar{D}^0\gamma K$ [171] revealed that the invariant mass recoiling against the Kaon displays a significant enhancement with a maximum at approximately 3875 MeV, which is only about 3 MeV above the $D^0\bar{D}^{*0}$ threshold. The observed events can all be in fact attributed to the process $B \rightarrow (D^0\bar{D}^{*0} + \bar{D}D^{*0})K$ since no distinction between the D^{*0} mesons and their decay products was done. Moreover, the yield of the heavy meson pairs within the above-threshold peak is about ten times larger than that of the $\pi^+\pi^-J/\psi$ and $\pi^+\pi^-\pi^0J/\psi$ channels at the peak of $X(3872)$. It has been most recently argued [193] that a very plausible explanation of the observed enhancement of the $D^0\bar{D}^{*0}$ production combined with the smaller observed $X(3872)$ peak in the $\pi^+\pi^-J/\psi$ channel is that both these phenomena are due to a virtual state [194, 195] in the $D^0\bar{D}^{*0}$ channel. In this picture the observed peak in the $\pi^+\pi^-J/\psi$ and $\pi^+\pi^-\pi^0J/\psi$ mass spectra is in fact a cusp with a sharp maximum at the $D^0\bar{D}^{*0}$ threshold.

The isospin properties of such near-threshold virtual state can be analyzed[196] within the approximation of small interaction radius. Such approach is similar to the ‘universal scattering length’ approximation[183], and differs in including the effect of the nearby threshold for charged mesons D^+D^{*-} . An interesting energy-dependent behavior of the isotopic properties arises from the mere fact of the mass splitting Δ between the two isospin-related and coupled $D\bar{D}^*$ channels. In particular it can be argued that the expected pattern of the isospin breaking is consistent with the observed relative yield of $\pi^+\pi^-J/\psi$ and $\pi^+\pi^-\pi^0J/\psi$ at the peak which experimentally [168] corresponds to $\mathcal{B}(X \rightarrow \pi^+\pi^-\pi^0J/\psi)/\mathcal{B}(X \rightarrow \pi^+\pi^-J/\psi) = 1.0 \pm 0.4 \pm 0.3$. Moreover, the production amplitude for the $I = 1$ state $\pi^+\pi^-J/\psi$ in the considered approximation necessarily has a zero between the $D^0\bar{D}^{*0}$ and D^+D^{*-} thresholds, thus reducing the apparent width of the cusp and putting it in line with the experimental limit [162] $\Gamma < 2.3$ MeV on the width of the peak in this particular channel.

The approximation of small interaction radius is applicable at small energy $E = M(D\bar{D}^*) - M(D^0) - M(D^{*0})$ for a consideration of the strong dynamics of two coupled channels $D^0\bar{D}^{*0} + \bar{D}D^{*0}$ and $D^+D^{*-} + D^-D^{*+}$, which for brevity can be called n and c channels. The energy range of interest for the discussion of the X peak is from few MeV below the n threshold and up to the c threshold, i.e. up to $E \approx \Delta = M(D^+D^{*-}) - M(D^0\bar{D}^{*0}) \approx 8.1$ MeV. In this range the scale of the c.m. momentum (real and virtual) in either channel is set by $\sqrt{2m_r\Delta} \approx 127$ MeV, where $m_r \approx 970$ MeV is the reduced mass for the meson pair. One can apply in this region of soft momenta the standard picture of the strong- interaction scattering (see e.g. in the textbook [197]), where the strong interaction is localized at distances $r < r_0$ such that $r_0\sqrt{2m_r\Delta}$ can be considered as a small parameter. Considering for definiteness an energy value between the two thresholds, $0 < E < \Delta$, one can write the corresponding wave functions (up to an overall normalization constant) as

$$\chi_n(r) = \sin(k_n r + \delta), \quad \chi_c(r) = \xi \exp(-\kappa_c r), \quad (141)$$

where $k_n = \sqrt{2m_rE}$ and $\kappa_c = \sqrt{2m_r(\Delta - E)}$, δ is the elastic⁹ scattering phase in the n channel and the constant ξ , generally energy- dependent, describes the relative normalization and phase of the wave function for the two channels.

The wave functions (141) should be matched at $r \approx r_0$ to the solution of the ‘inner’ problem, i.e. that in the region of the strong interaction. In the limit of small r_0 all the complexity of the ‘inner’ problem reduces to only two parameters. Namely, in the region of the strong interaction the n and c channels are not independent and get mixed. Due to the isotopic symmetry of the strong interaction

⁹In this consideration the small inelasticity due to the $\pi^+\pi^-J/\psi$ and $\pi^+\pi^-\pi^0J/\psi$ channels is neglected and will be included later. Also the small width of the D^* mesons is entirely neglected. The effects of the latter width are considered in the most recent papers [198, 199].

the independent are the channels with definite isospin, $I = 0$ and $I = 1$, corresponding to the functions $\chi_0 = \chi_n + \chi_c$ and $\chi_1 = \chi_n - \chi_c$, and the matching parameters are the logarithmic derivatives $-\kappa_0$ and $-\kappa_1$ of these functions at $r = r_0$. Using the assumption of small r_0 the matching condition for the functions from Eq.(141) can be shifted to $r = 0$, so that one can write the resulting matching equations as

$$\frac{k_n \cos \delta - \xi \kappa_c}{\sin \delta + \xi} = -\kappa_0 , \quad \frac{k_n \cos \delta + \xi \kappa_c}{\sin \delta - \xi} = -\kappa_1 . \quad (142)$$

These equations determine both the scattering phase δ and the constant ξ as

$$\cot \delta = -\frac{\kappa_{\text{eff}}}{k_n} \quad (143)$$

with

$$\kappa_{\text{eff}} = \frac{2\kappa_0\kappa_1 - \kappa_c\kappa_1 - \kappa_c\kappa_0}{\kappa_0 + \kappa_1 - 2\kappa_c} , \quad (144)$$

and

$$\xi = \frac{\kappa_0 - \kappa_1}{2\kappa_c - \kappa_1 - \kappa_0} \sin \delta . \quad (145)$$

The nonrelativistic scattering amplitude in the n channel is therefore given by [197]

$$F = -\frac{1}{\kappa_{\text{eff}} + i k_n} , \quad (146)$$

and the scattering length a is thus found from the $E = 0$ limit of this expression as

$$a = \left. \frac{1}{\kappa_{\text{eff}}} \right|_{E=0} = \frac{\kappa_0 + \kappa_1 - 2\sqrt{2m_r\Delta}}{2\kappa_0\kappa_1 - (\kappa_0 + \kappa_1)\sqrt{2m_r\Delta}} . \quad (147)$$

The whole approach is applicable if the scattering length is large in the scale of strong interaction. A large positive value of a implies an existence of a shallow bound state, while a large negative a corresponds to the situation with a virtual state [197]. According to the estimates of Ref.[193] the required by the data scattering length in the problem considered is $-(3 \div 4)$ fm, corresponding to a negative and quite small indeed parameter $\kappa_{\text{eff}}(E = 0) \approx (50 \div 60)$ MeV.

The physical picture, consistent with a small κ_{eff} , and which could be argued on general grounds [155], is that an attraction in the $I = 0$ channel is strong enough to provide a small value of κ_0 , while the interaction in the $I = 1$ channel is either a weak attraction or, more likely, a repulsion. In both cases the absolute value of κ_1 is large, i.e. of a normal strong interaction scale, with the sign being respectively negative or positive. Another, purely phenomenological, argument in favor of large $|\kappa_1|$ is that no peculiar near-threshold behavior is observed in the production of the $I = 1$ charged states, e.g. $D^0 D^{*-}$. At large $|\kappa_1|$ the expression (144) simplifies and takes the approximate form

$$\kappa_{\text{eff}} \approx 2\kappa_0 - \kappa_c . \quad (148)$$

Using this approximation, one can readily see that in order for $\kappa_{\text{eff}}(E = 0)$ to be negative and small, the parameter κ_0 has to be positive and quite small:

$$\kappa_0 < \sqrt{m_r\Delta/2} \approx 63 \text{ MeV}. \quad (149)$$

It is interesting to note that in the discussed picture the interaction in the $I = 0$ state is strong enough by itself to produce a shallow bound state in the limit of exact isospin symmetry, i.e. at $\Delta \rightarrow 0$. In reality the isospin breaking by the mass difference between the charged and neutral charmed mesons turns out to be sufficiently significant to deform the bound state into a virtual one, i.e. to shift the

pole of the scattering amplitude from the first sheet to the second sheet of the Riemann surface for the amplitude as a complex function of the energy E .

The inelasticity in the n and c channels, related to decays to the observed final states $\pi^+\pi^-J/\psi$ ($\rho J/\psi$), $\pi^+\pi^-\pi^0J/\psi$ ($\omega J/\psi$), $\gamma J/\psi$ and probably other, appears to be reasonably small, as one can infer from the observed [170, 171] dominance of the $D^0\bar{D}^{*0}$ production in the threshold region, and can be parametrized by a small imaginary shift $i\gamma$ of the denominator of the scattering amplitude in Eq.(146):

$$F = -\frac{1}{\kappa_{\text{eff}} + i k_n + i \gamma} \approx -\frac{1}{2\kappa_0 - \kappa_c + i k_n + i \gamma}. \quad (150)$$

If one further assumes [180, 193] that the ‘seed’ decay $B \rightarrow XK$ is a short-distance process, one would find that the yield in each final channel coupled to X is proportional to that channel’s contribution to the unitary cut of the amplitude F . This implies in particular that

$$\mathcal{B}[B \rightarrow (D^0\bar{D}^{*0} + \bar{D}D^{*0})K] : \mathcal{B}(B \rightarrow \omega J/\psi K) : \mathcal{B}(B \rightarrow \rho J/\psi K) = k_n |F|^2 : \gamma_\omega |F|^2 : \gamma_\rho |F|^2, \quad (151)$$

where the specific expression for $|F|^2$ depends on the value of the energy E relative to the n and c thresholds, as given by Eq.(150) an its analytical continuation across the thresholds. Besides the energy dependence of the overall factor $|F|^2$, the heavy meson channel contains the phase space factor k_n , while for the $\omega J/\psi$ and $\rho J/\psi$ yields an additional dependence on the energy arises from the factors γ_ω and γ_ρ .

A certain variation of the width parameter γ_ω for the $\pi^+\pi^-\pi^0J/\psi$ channel in the discussed range of energy is of a well known kinematical origin. Indeed, the central value of the mass of the ω resonance puts the threshold for the channel $\omega J/\psi$ at 3878.5 MeV, which corresponds to $E \approx 6.7$ MeV in our conventions, i.e. squarely between the n and c thresholds. Any production of the $\pi^+\pi^-\pi^0J/\psi$ states at smaller invariant mass is a sub-threshold process, possible due to the width Γ_ω of the ω resonance. In other words, the energy dependence of the width factor γ_ω can be estimated as

$$\gamma_\omega = |A_\omega|^2 q_{\text{eff}}^{(\omega)}, \quad (152)$$

where A_ω is the amplitude factor for the coupling to the $\omega J/\psi$ channel and $q_{\text{eff}}^{(\omega)}$ is the effective momentum of ω at the invariant mass M calculated as

$$q_{\text{eff}}^{(\omega)}(M) = \int_{m_0}^{M-m_{J/\psi}} |\vec{q}(m)| \frac{m_\omega \Gamma_\omega}{(m^2 - m_\omega^2)^2 + m_\omega^2 \Gamma_\omega^2} \frac{dm^2}{\pi} \quad (153)$$

with the c.m. momentum $|\vec{q}(m)|$ found in the standard way:

$$|\vec{q}(m)| = \frac{\sqrt{[(M - m_{J/\psi})^2 - m^2][(M + m_{J/\psi})^2 - m^2]}}{2M}. \quad (154)$$

The lower limit m_0 in the integral in Eq.(153) can be chosen anywhere sufficiently below $m_\omega - \Gamma_\omega$, since the Breit-Wigner curve in the integrand rapidly falls off away from the resonance.

Numerically, the effective momentum $q_{\text{eff}}^{(\omega)}$ can be estimated as varying from approximately 20 MeV to 50 MeV between the n and c thresholds, i.e. when E changes from $E = 0$ to $E = \Delta$. In the $\rho J/\psi$ channel the expected energy behavior of the yield is quite different. If one writes the corresponding width factor γ_ρ similarly to Eq.(152) as

$$\gamma_\rho = |A_\rho|^2 q_{\text{eff}}^{(\rho)}, \quad (155)$$

the effective momentum $q_{\text{eff}}^{(\rho)}$ can be estimated as varying only slightly due to the large width of the ρ resonance: $q_{\text{eff}}^{(\rho)} \approx (125 \div 135)$ MeV as the energy changes between $E = 0$ and $E = \Delta$.

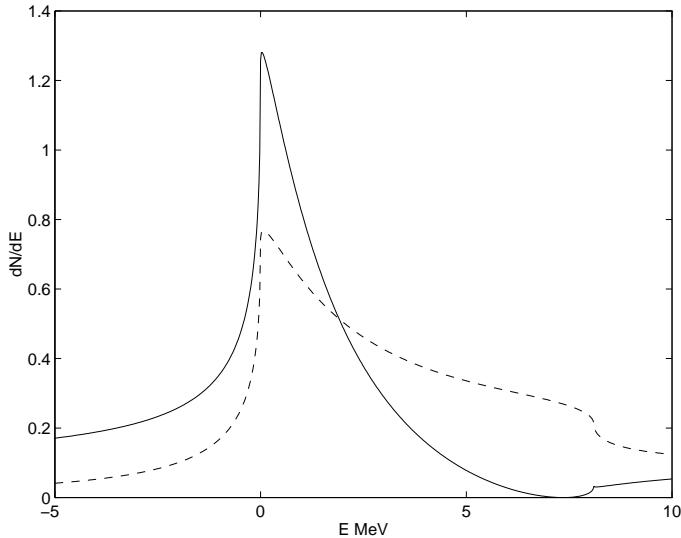


Figure 9: The expected shape (in arbitrary units) of the virtual state peak in the yield of $\pi^+\pi^-J/\psi$ (solid) and $\pi^+\pi^-\pi^0J/\psi$ (dashed) channels.

The amplitudes A_ω and A_ρ on the other hand should display a noticeably different behavior in the energy range of interest, due to the rapid (and different from each other) variation of the $I = 0$ and $I = 1$ scattering amplitudes of the meson-meson scattering. Namely assuming that the amplitudes for production of the $I = 0$ and $I = 1$ at short distances are described by constant (in the considered energy range) factors Φ_ω and Φ_ρ and that the effective strong production radius is R , one can arrive[196], after taking into account the rescattering of mesons, at the following estimate for the ratio of the amplitudes

$$\frac{A_\rho}{A_\omega} = \frac{\kappa_0 - \kappa_c}{\kappa_1 - \kappa_c} (1 - \kappa_1 R) \frac{\Phi_\rho}{\Phi_\omega}. \quad (156)$$

Since the situation where the X peak is a virtual state corresponds to a small positive κ_0 satisfying the condition (149), the amplitude A_ρ described by Eq.(156) should necessarily change sign between the n threshold, where $\kappa_c = \sqrt{2m_r\Delta}$, and the c threshold, where $\kappa_c = 0$.

The expected difference in the shape of the cusp in the $\rho J/\psi$ and $\omega J/\psi$ channels is illustrated in Fig.9. In these plots the parameters of the virtual state correspond to the scattering length $a = -(4 + 0.5 i)$ fm, which is close to the possible fit values of the scattering length found in Ref.[193]. In the limit of large κ_1 this value of a translates into $\kappa_0 \approx 38$ MeV and $\gamma \approx 6$ MeV. One can see from Fig.9 that due to the discussed zero of the amplitude, the peak in the $\pi^+\pi^-J/\psi$ channel is expected to be quite narrow in agreement with the experimental limit on the width of $X(3872)$. The plots in Fig.9 are normalized to the same total yield in each channel over the shown energy range in order to approximate the experimentally observed relative yield. Such normalization corresponds to setting

$$\left| \frac{\kappa_1}{1 - \kappa_1 R} \frac{\Phi_\omega}{\Phi_\rho} \right| \approx 175 \text{ MeV},$$

which value does not appear to be abnormal, even though at present we have no means of independently estimating this quantity.

Summarizing the situation with the $X(3872)$ resonance it can be stated that as of the time of this writing, full four years after its discovery and even though this resonance is listed among established particles in the Tables[27], its real status is still lively debated in the literature. It is clear that this peak is very closely related to the near-threshold dynamics of the $D\bar{D}^*$ meson pairs, but it is still

unclear whether this is a shallow bound molecular state, or a virtual state, or something else. It may well be that a further pursuit of the puzzles arising in connection with the $X(3872)$ peak may provide important clues to understanding multi-quark dynamics.

6.3 Higher $J^{PC} = 1^{--}$ Resonances

The existence of reasonably broad resonances above the open charm threshold has been fully expected on the basis of potential models[15]. What has not been fully expected however are the peculiar properties of individual resonances, different for different states, and never failing to bring unexpected surprises.

6.3.1 The Vicinity of $\psi(4040)$

An unusual behavior of the cross section of e^+e^- annihilation into charmed meson pairs in the energy region, which is now associated with the resonance $\psi(4040)$, was first pointed out in Ref.[179]. Namely, it was noticed that the production of the $D^*\bar{D}^*$ is greatly enhanced relative to $D\bar{D}$ and $D\bar{D}^*$ in comparison with a simple spin-model estimate[179, 15]. The cross section in each channel is proportional to the P wave factor p^3 with p being the c.m. momentum in the corresponding final state. The model predicted the ratio 1:3:7 of the extra factors on top of the p^3 in the cross section for production of respectively $D\bar{D}$, $D\bar{D}^* + \bar{D}D^*$ and $D^*\bar{D}^*$. The then existing data however indicated that the cross section for production of the pairs of vector mesons $D^*\bar{D}^*$ near their threshold was enhanced by a factor of tens to hundreds in comparison with this estimate. This observation gave the reason for the suggestion[179] that $\psi(4040)$ is in fact a molecular state made of the vector mesons. However a more conventional potential model[15] could well accommodate the $\psi(4040)$ peak as a dominantly 3^3S_1 state, and it was also argued[200] that the suppression of the decay modes for this resonance with one or two pseudoscalar mesons is due to an ‘accidental’ kinematical zero of the overlap integrals in a model of such decay.

A significantly more detailed, than previously available, data on the production of charmed meson pairs have been accumulated recently[187, 188, 201] and the new data also include the cross section for production of pairs of strange charmed mesons $D_s\bar{D}_s$. The new data still indicate an unusually strong production of the vector mesons, $D^*\bar{D}^*$, near their threshold and also reveal intricate features of the behavior of the cross section in other channels as the energy sweeps across the $D^*\bar{D}^*$ threshold(s) and the $\psi(4040)$ resonance, which definitely points at a strong coupling between the channels. This behavior is more illustrative in terms of dimensionless rate coefficients R_i defined as follows[202]

$$\begin{aligned}\sigma(e^+e^- \rightarrow D\bar{D}) &= \sigma_0(s) 2 v_D^3 R_1 , & \sigma(e^+e^- \rightarrow D_s\bar{D}_s) &= \sigma_0(s) v_{D_s}^3 R_2 , \\ \sigma(e^+e^- \rightarrow D\bar{D}^* + D^*\bar{D}) &= \sigma_0(s) 6 \left(\frac{2p}{\sqrt{s}}\right)^3 R_3 , & \sigma(e^+e^- \rightarrow D^*\bar{D}^*) &= \sigma_0(s) 7 (v_0^3 + v_+^3) R_4 ,\end{aligned}\quad (157)$$

with $\sigma_0 = \pi \alpha^2 / (3s)$. Here v_D , v_{D_s} , v_0 and v_+ stand for the c.m. velocities of each of the mesons in respectively the channels $D\bar{D}$, $D_s\bar{D}_s$, $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$, while for the channel $D\bar{D}^* + D^*\bar{D}$ with mesons of unequal mass the velocity factor is replaced by $(2p/\sqrt{s})$ with p being the c.m. momentum. The values of R_i calculated from a preliminary version of the data[187] are shown as data points in Fig.10. The extra factors 1, 3 and 7 in Eq.(157) for respectively the pseudoscalar-pseudoscalar, pseudoscalar-vector and the vector-vector channels correspond to the ratio of the corresponding production cross section in the simplest model[179, 15] of independent quark spins, so that the inequality between R_1 , R_3 and R_4 also illustrates a conspicuous deviation from this model. In particular the very large values of R_4 describe the unusually strong enhancement of the vector-vector channel.

The curves in the plots of Fig.10 correspond to a fit[202] with one resonance and a non-resonant background with coupling among the channels and with a proper treatment of the onset of the inelasticity at the two thresholds for the vector meson pairs, $D^{*0}\bar{D}^{*0}$ at 4013 MeV and $D^{*+}\bar{D}^{*-}$ at 4020 MeV.

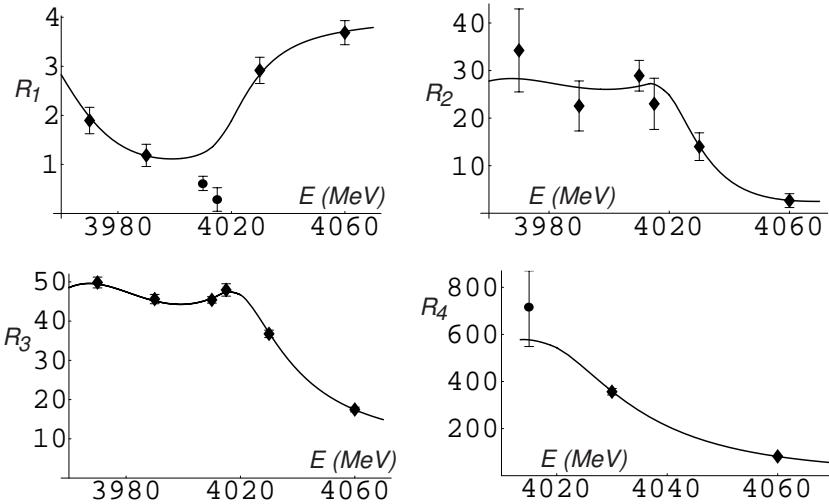


Figure 10: The plots of the rate coefficients R corresponding to the fit with excluded data points at energy 4010 and 4015 MeV for the $D\bar{D}$ channel and at 4015 MeV for the $D^*\bar{D}^*$ channel. The excluded points are shown by filled circles.

A fit to the full set of data between 3970 MeV and 4060 MeV turns out to be impossible with an acceptable χ^2 (the best fit corresponds to $\chi^2/NDF = 17.6/8$. The offending are the data points at 4010 and 4015 MeV for the $D\bar{D}$ channel and, to some extent, the data point at 4015 MeV for the $D^*\bar{D}^*$ production. If those points are removed, a fit to the data can be found with $\chi^2/NDF = 3.0/5$ and the resulting central values of the resonance nominal mass, width and Γ_{ee} are $M = 4019$ MeV, $\Gamma = 65$ MeV and $\Gamma_{ee} = 1.7$ keV. A more detailed analysis[203] of the final data including the radiative corrections resulted in the values $M = 4013 \pm 4$ MeV, $\Gamma = 66 \pm 8$ MeV and $\Gamma_{ee} = 1.9 \pm 0.7$ keV.

The peculiar behavior of the $D\bar{D}$ points at 4010 - 4015 MeV may indicate a presence of another narrow resonance at this energy at or between the $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ thresholds, which has a quite small coupling to e^+e^- , corresponding to Γ_{ee} in the range of a few tenths of eV[202]. Such resonance could then be explained as a P wave $D^*\bar{D}^*$ molecule which is expected[155] to have the e^+e^- width in the same ballpark.

6.3.2 $\psi(4170)$

The main surprise of the peak $\psi(4170)$ (also labeled as $\psi(4160)$ in the Tables [27]) is that it corresponds to the absolute maximum of the cross section for production in e^+e^- annihilation of strange charmed meson pairs $D_s\bar{D}_s^* + \bar{D}_s D_s^*$. The cross section at the maximum reaches[187, 188] nearly 1 nb, which is a very large value for an exclusive state with hidden strangeness and charm. This property of the peak is intensively used in the experimental studies of the D_s mesons, but it has no theoretical explanation.

6.3.3 $Y(4260)$

The total cross section for charm production in e^+e^- annihilation has a well known[27] deep minimum around the c.m. energy 4260 MeV. It is at about the same energy where the BaBar experiment, while performing a survey of the region around 4 GeV in the radiation return process $e^+e^- \rightarrow \gamma X$, found[204] a peak in the cross section for the *exclusive* final state $\pi^+\pi^-J/\psi$. The existence of the peak was further confirmed by Belle[205] and by CLEO[206] using the same radiative return method and also by CLEO[207] by a direct scan of the e^+e^- annihilation in the peak region. The peak at the same invariant mass is also indicated by the data[182] on the decays $B \rightarrow \pi^+\pi^-J/\psi K$. The values for the mass and

the width measured by independent experiments are in a statistical agreement with each other, and the average values of these parameters according to the updated Tables[27] are $M(Y) = 4264^{+10}_{-12}$ MeV and $\Gamma(Y) = 83^{+20}_{-17}$ MeV.

The only so far observed decays channels of $Y(4260)$ are $\pi^+\pi^-J/\psi$, $\pi^0\pi^0J/\psi$ (in a fair agreement with the isospin of Y being equal to zero) and K^+K^-J/ψ . Notably, the data show no peak in any channels with D meson pairs. In particular the most stringent upper limit is found[208] for the $D\bar{D}$ channel: $\Gamma(Y \rightarrow D\bar{D})/\Gamma(Y \rightarrow \pi^+\pi^-J/\psi) < 1.0$ at 90% confidence level. Such behavior is next to impossible to explain by considering the resonance as a charmonium state even though the mass of Y is close to the expectation for the $4S$ state[209], since lower $J^{PC} = 1^{--}$ resonances $\psi(3770)$, $\psi(4040)$ decay practically exclusively to D meson pairs. A whole spectrum of interpretations of the state $Y(4260)$ has been suggested[210] including $c\bar{c} + glue$ hybrids[211, 212] and a tetraquark $c\bar{s}c\bar{s}$ state assignment[213], which does not look very promising, given that the ratio of the decay rates[206] $\Gamma(Y \rightarrow K^+K^-J/\psi)/\Gamma(Y \rightarrow \pi^+\pi^-J/\psi) \approx 0.15$ does not show any enhanced presence of strangeness within $Y(4260)$.

6.3.4 $J^{PC} = 1^{--}$ Peaks in $\pi^+\pi^-\psi'$

Subsequent studies of the radiative return events have lead to an observation by BaBar[214] of a ‘broad structure’ in the final state $\pi^+\pi^-\psi'$. A single resonance fit to the data yielded the mass of $M = 4324 \pm 24$ MeV and the width of $\Gamma = 172 \pm 33$ MeV. A further investigation of this final channel resulted in an observation by Belle[215] of a peak with the mass of $4361 \pm 9 \pm 9$ MeV and the width of $74 \pm 15 \pm 10$ MeV, possibly compatible with the structure observed by BaBar, and an *additional* narrower peak at $4664 \pm 11 \pm 5$ MeV with the width of $48 \pm 15 \pm 3$ MeV.

6.3.5 $Z(4430)$ and Remarks on New States

Most recently the Belle experiment presented data[216] on observation of a peak $Z(4430)$ in the charged system $\pi^\pm\psi'$ emerging from the decays $B \rightarrow K\pi^\pm\psi'$. The parameters of the peak are $M = 4433 \pm 4 \pm 2$ MeV, $\Gamma = 45^{+18}_{-15}{}^{+30}_{-13}$ MeV. The statistical significance of the observation corresponds to 6.5σ , however in view of an utmost importance of the observed peak an additional confirmation is eagerly awaited. Unlike the previously discussed electrically neutral states, which all at least had a chance of being a pure charmonium, this one is charged and has isospin $I = 1$ and clearly cannot be a pure $c\bar{c}$ but has to contain light quarks in addition to the $c\bar{c}$ pair. Naturally, a variety of interpretations of $Z(4430)$ has been suggested: a threshold peak[217, 218] or a resonance[219] a loosely bound molecular state[220] in the $D^*\bar{D}_1(2420)$ meson system, a radially excited tetraquark[221], a QCD-string based model[222], and a baryonium state[223]. In either case, whether or not the peak $Z(4430)$ is related to the $D^*\bar{D}_1(2420)$ meson system, its ‘affinity’ to the particular decay channel $\pi\psi'$, rather than the multitude of available channels with D mesons presents a very intriguing riddle.

Clearly, a similar riddle also relates to the states $Y(4260)$, and the ‘structures’ in the $\pi^+\pi^-\psi'$ channel at 4.32 and 4.66 GeV. Perhaps, the simplest explanation of such unusual prominence in the decays of each state of a particular charmonium level accompanied by light hadrons would be a picture, where a charmonium state, e.g. J/ψ , or ψ' is ‘stuck’ in a light hadronic state. In a sense, such picture can be viewed as that of a bound state of a relatively compact charmonium inside a light hadron having a larger spatial size. This possibility in fact returns us to the previous discussion of existence of bound states of J/ψ and/or ψ' in light nucleons. It may well be that some of the recently found high mass states are in fact such bound states in mesonic rather than baryonic matter. If this indeed is the case, one can naturally expect an existence of similar baryonic states, e.g. bound states of either J/ψ or ψ' ‘inside’ a proton, or even ‘inside’ a deuteron. Needless to mention that an observation of such baryonic states would be of an immense interest.

7 Summary

The potential models of interaction within charmonium generally agree with the lower-mass part of the observed spectrum of the resonances, where two long-missing states, the ${}^1P_1 h_c$ and the ${}^2S_0 \eta'_c$, have eventually been located. In the mass region at and above the threshold for charmed mesons their dynamics apparently plays an important role in determining the spectrum of states. At lower masses some fine effects of the interaction between the quark and the antiquark still remain unsolved, such as the behavior of the spin-dependent forces, describing the splittings of the χ_{cJ} states, and the spin-spin interaction giving rise to the small splitting between the c.o.g. of the χ_{cJ} resonances and the spin-singlet h_c .

Less model-dependent and based on the underlying theory of QCD spectral methods for analyzing the charmonium states, either analytical, or numerical lattice simulations, reproduce reasonably well the properties of the lowest state in each channel. However, these methods are intrinsically less appropriate for handling radially excited states, and they also eventually run into limitations discussed in the Section 2.3.2.

The overall picture of the hidden charm decay through strong or electromagnetic annihilation is in agreement with the data, still leaving us with a number of puzzles in some particular cases, by solving which we might gain some new insights into hadron dynamics. Among such puzzles are the larger than expected total widths of η_c and χ_{c0} and the non-similarity of some exclusive decay channels for J/ψ and ψ' - a violation of the ‘12% rule’. These yet to be explained properties of the well known charmonium states very likely indicate a presence of substantial nonperturbative effects in the annihilation processes.

The radiative transitions between charmonium levels are in line with generic considerations based on nonrelativistic quantum mechanics and also with the results of specific potential models. A notable exception is the $M1$ transition $J/\psi \rightarrow \gamma \eta_c$, which is significantly weaker than any theoretical estimates. Coupled with the large total width of η_c this may signal a mixing of the η_c with light $J^{PC} = 0^{-+}$ degrees of freedom.

The hadronic transitions between charmonium resonances offer a window into the interaction of the charmonium states with soft gluon fields, and also into the details of the conversion of soft gluons to light mesons. The former interaction is described by the multipole expansion in QCD, while the latter conversion is tractable due to the chiral algebra and low-energy theorems in QCD based on the conformal and axial anomalies. Such approach turned out to be successful in describing the transitions $\psi' \rightarrow \pi\pi J/\psi$ and $\psi' \rightarrow \eta J/\psi$ and in predicting finer details such as the small D wave in the former transition. Due to this success it can be believed that the recently established discrepancy by a factor of about 1.5 between the observed and the theoretical rate of the decay $\psi' \rightarrow \pi^0 J/\psi$ is not a result of a failure of the multipole expansion and the low-energy theorems, but rather very likely signals a presence of a small four-quark isovector component in the ψ' resonance.

The knowledge of the strength of the interaction of charmonium with soft gluon fields, the chromo-polarizability, can be applied to considering the behavior of charmonium inside a hadronic media, such as the interaction of slow charmonium with nucleons. The estimates of the chromo-polarizability from the rate of the decay $\psi' \rightarrow \pi\pi J/\psi$ indicate that slow charmonium interacts quite strongly with light hadrons. This interaction may well result in existence of bound states of charmonium in light nuclei or with ordinary hadronic resonances.

The unusual new states at and above charmed meson thresholds keep mushrooming in the most recent experimental data suggesting that this mass region is a real playground for much of the exotics that has been previously speculated about: tetraquarks, hybrids, molecules, and mixtures of those. The peak at $X(3872)$ that started it all still offers intriguing puzzles, and further studies are required in order to conclusively assess its internal structure and even its very status as of a resonance. The newly found peaks in the $\pi\pi J/\psi$ and $\pi\pi\psi'$ invariant mass spectra and especially the latest peak $Z(4430)$ in the $\pi^\pm\psi'$ channel lead us further down the path of acceptance of multiquark hadronic states being as

commonplace, as are nuclei if viewed as multiquark systems.

Many of the properties of charmonium, starting with a very small width of the J/ψ resonance, came as a great surprise when first observed. Understanding these properties and in some cases making their description routine, has greatly advanced the knowledge of the dynamics of quarks and gluons. Now, thirty three years after the discovery of J/ψ , the studies of charmonium keep bringing new surprises and puzzles, solving which will hopefully result in further advances.

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References

- [1] J. J. Aubert *et.al.*, *Phys. Rev. Lett.* **33**, 1404 (1974).
- [2] J. E. Augustin *et.al.*, *Phys. Rev. Lett.* **33**, 1406 (1974).
- [3] G. S. Abrams *et.al.*, *Phys. Rev. Lett.* **33**, 1453 (1974).
- [4] V. A. Novikov *et. al.*, *Phys. Rep.* **41C**, 1 (1978).
- [5] N. Brambilla *et al.* [Quarkonium Working Group], Report FERMILAB-FN-0779, CERN-2005-005, Dec 2004, arXiv:hep-ph/0412158.
- [6] E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Report CLNS-07-1988, EFI-06-15, FERMILAB-PUB-07-006-T, Jan 2007, arXiv:hep-ph/0701208.
- [7] M. Peter, *Phys. Rev. Lett.* **78**, 602 (1997). *Nucl. Phys. B* **501**, 471 (1997).
- [8] Y. Schroder, *Phys. Lett.* **B447**, 321 (1999).
- [9] N. Brambilla, A. Pineda, J. Soto and A. Vairo, *Phys. Rev. D* **60**, 091502 (1999).
- [10] K. G. Wilson, *Phys. Rev. D* **10**, 2445 (1974).
- [11] G. S. Bali, *Phys. Rep.* **343**, 1 (2001).
- [12] M. B. Voloshin, *Nucl. Phys. B* **154**, 365 (1979).
- [13] M. B. Voloshin, *Sov. J. Nucl. Phys.* **36**, 143 (1982) [*Yad. Fiz.* **36**, 247 (1982)].
- [14] H. Leutwyler, *Phys. Lett.* **B98**, 447 (1981).
- [15] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, *Phys. Rev. Lett.* **36**, 500 (1976); *K. D. Lane and E. Eichten*, *Phys. Rev. Lett.* **37**, 477 (1976) [Erratum-*ibid.* **37**, 1105 (1976)]. E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, *Phys. Rev. D* **17**, 3090 (1978) [Erratum-*ibid. D* **21**, 313 (1980)]; E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, *Phys. Rev. D* **21**, 203 (1980).
- [16] J. L. Richardson, *Phys. Lett.* **B82**, 272 (1979).
- [17] W. Buchmuller and S. H. H. Tye, *Phys. Rev. D* **24**, 132 (1981).
- [18] E. J. Eichten, K. Lane, and C. Quigg, *Phys. Rev. Lett.* **89**, 162002 (2002); *Phys. Rev. D* **69**, 094019 (2004).
- [19] E. J. Eichten, K. Lane, and C. Quigg, *Phys. Rev. D* **73**, 014014 (2006) [Erratum *ibid.* 079903 (2006)].
- [20] A. Martin, Report CERN-PH-TH-2007-069, May 2007, arXiv:0705.2353 [hep-ph].

- [21] J. Pumplin, W. Repko and A. Sato, *Phys. Rev. Lett.* **35**, 1538 (1975).
- [22] H. J. Schnitzer, *Phys. Rev. Lett.* **35**, 1540 (1975).
- [23] D. Ebert, R. N. Faustov and V. O. Galkin, *Phys. Rev. D* **67**, 014027 (2003).
- [24] V.B. Berestetskii, E.M. Lifshits and L.P. Pitaevskii, *Quantum Electrodynamics*, Pergamon 1982.
- [25] W. Buchmuller, Y. J. Ng and S. H. H. Tye, *Phys. Rev. D* **24**, 3003 (1981).
- [26] M. B. Voloshin, *Sov. J. Nucl. Phys.* **35**, 592 (1982) [*Yad. Fiz.* **35**, 1016 (1982)].
- [27] W.M. Yao *et al.* [Particle Data Group], *J.Phys. G* **33**, 1 (2006) and 2007 online update.
- [28] T. A. Armstrong *et al.*, *Phys. Rev. Lett.* **69**, 2337 (1992).
- [29] M. Andreotti *et al.*, *Phys. Rev. D* **72**, 032001 (2005).
- [30] J. L. Rosner *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **95**, 102003 (2005).
- [31] K.K. Seth, Talk at the Quarkonium Working Group Meeting, DESY, Hamburg, Oct. 17-20, 2007; URL: http://www.desy.de/qwg07/talks/Seth_071019.pdf
- [32] S. Godfrey and N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [33] L. P. Fulcher, *Phys. Rev. D* **44**, 2079 (1991).
- [34] J. Zeng, J. W. Van Orden and W. Roberts, *Phys. Rev. D* **52**, 5229 (1995).
- [35] S. Uehara *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **96**, 082003 (2006).
- [36] K. Abe *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **94**, 182002 (2005).
- [37] B. Aubert *et al.* [BABAR Collaboration], arXiv:0711.2047 [hep-ex]
- [38] F. E. Close, arXiv:0706.2709 [hep-ph]
- [39] T. Barnes, *Int. J. Mod. Phys. A* **21**, 5583 (2006).
- [40] K. Abe *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **98**, 082001 (2007).
- [41] V. A. Novikov *et.al.*, *Phys. Rev. Lett.* **38**, 626 (1977) [Erratum-*ibid.* **38**, 791 (1977)].
- [42] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, *Phys. Lett. B* **77**, 80 (1978).
- [43] A. I. Vainshtein, V. I. Zakharov and M. A. Shifman, *JETP Lett.* **27**, 55 (1978) [*Pisma Zh. Eksp. Teor. Fiz.* **27**, 60 (1978)].
- [44] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979).
- [45] K. G. Chetyrkin, J. H. Kühn and M. Steinhauser, *Phys. Lett. B* **371**, 93 (1996); *Nucl. Phys. B* **505**, 40 (1997).
- [46] K. G. Chetyrkin, J. H. Kühn and C. Sturm, *Eur. Phys. J. C* **48**, 107 (2006).
- [47] R. Boughezal, M. Czakon and T. Schutzmeier, *Phys. Rev. D* **74**, 074006 (2006).
- [48] V. A. Ilyin, V. A. Smirnov, A. Y. Taranov and K. G. Chetyrkin, *Sov. J. Nucl. Phys.* **49**, 939 (1989) [*Yad. Fiz.* **49**, 1515 (1989)].
- [49] J. Z. Bai *et al.* [BES Collaboration], *Phys. Rev. Lett.* **88**, 101802 (2002).
- [50] M. Ablikim *et al.*, [BES Collaboration], arXiv:hep-ex/0612054.
- [51] J. H. K'uhn, M. Steinhauser and C. Sturm, arXiv:0705.2335 [hep-ph].
- [52] S. Okubo, *Phys. Lett* **5**, 165 (1963).
- [53] G. Zweig, CERN Reports TH 401 and TH 412 (1964).
- [54] J. Iizuka, *Progr. Ther. Phys. Suppl.* **37**, 21 (1966).
- [55] S. Groote and A. A. Pivovarov, *JETP Lett.* **75**, 221 (2002) [*Pisma Zh. Eksp. Teor. Fiz.* **75**, 267 (2002)].
- [56] S. Groote and A. A. Pivovarov, *Eur. Phys. J. C* **21**, 133 (2001).
- [57] L. D .Landau, *Doklady Akademii Nauk SSSR*, 207 (1948).

- [58] C. N. Yang, *Phys. Rev.* **77**, 242 (1950).
- [59] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **191**, 301 (1981).
- [60] A. Pineda and F. J. Yndurain, *Phys. Rev. D* **58**, 094022 (1998) and *Phys. Rev. D* **61**, 077505 (2000).
- [61] Z. Li *et al.* [CLEO Collaboration], *Phys. Rev. D* **71**, 111103 (2005).
- [62] J. L. Rosner, *Phys. Rev. D* **64**, 094002 (2001).
- [63] J. L. Rosner, *Annals Phys.* **319**, 1 (2005).
- [64] R. Barbieri, R. Gatto and R. Kögerler, *Phys. Lett. B* **60**, 183 (1976).
- [65] W. Kwong, P. B. Mackenzie, R. Rosenfeld and J. L. Rosner, *Phys. Rev. D* **37**, 3210 (1988).
- [66] R. Barbieri, E. d'Emilio, G. Curci and E. Remiddi, *Nucl. Phys. B* **154**, 535 (1979).
- [67] I. Harris and L. M. Brown, *Phys. Rev.* **105**, 1656 (1957).
- [68] D. M. Asner *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **92**, 142001 (2004).
- [69] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. Lett.* **92**, 142002 (2004).
- [70] V. A. Novikov *et al.*, *Phys. Lett. B* **67**, 409 (1977).
- [71] R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, *Nucl. Phys. B* **192**, 61 (1981).
- [72] H. Mahlke, Talk at the Quarkonium Working Group Meeting, DESY, Hamburg, Oct. 17-20, 2007; URL: http://www.desy.de/qwg07/talks/Mahlke_071018.pdf
- [73] A. Ore and J. L. Powell, *Phys. Rev.* **75**, 1696 (1949).
- [74] W. E. Caswell, G. P. Lepage and J. R. Sapirstein, *Phys. Rev. Lett.* **38**, 488 (1977).
- [75] T. Appelquist and H. D. Politzer, *Phys. Rev. Lett.* **34**, 43 (1975).
- [76] P. B. Mackenzie and G. P. Lepage, *Phys. Rev. Lett.* **47**, 1244 (1981).
- [77] D. L. Scharre *et al.* [Mark II Collaboration], *Phys. Rev. D* **23**, 43 (1981).
- [78] N. E. Adam *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **94**, 232002 (2005).
- [79] R. A. Briere *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **95**, 062001 (2005).
- [80] M. Chanowitz, *Phys. Rev. D* **12**, 918 (1975).
- [81] D. Besson *et al.* [CLEO Collaboration], *Phys. Rev. D* **74**, 012003 (2006).
- [82] S. Menke, in *Proc. of the e^+e^- Physics at Intermediate Energies Conference, SLAC, Stanford, California, 30 Apr - 2 May 2001*, ed. Diego Bettoni, pp M09 [arXiv:hep-ex/0106011].
- [83] R. Barbieri, R. Gatto and E. Remiddi, *Phys. Lett. B* **61**, 465 (1976).
- [84] G. T. Bodwin, E. Braaten and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995) [Erratum *ibid. D* **55**, 5853 (1997)].
- [85] M. B. Voloshin, *Sov. J. Nucl. Phys.* **40**, 662 (1984) [*Yad. Fiz.* **40**, 1039 (1984)].
- [86] J. D. Jackson, *Phys. Rev. Lett.* **37**, 1107 (1976).
- [87] H. Grotch, D. A. Owen and K. J. Sebastian, *Phys. Rev. D* **30**, 1924 (1984).
- [88] K. J. Sebastian, H. Grotch and F. L. Ridener, *Phys. Rev. D* **45**, 3163 (1992).
- [89] M. Ambrogiani *et al.* [E835 Collaboration], *Phys. Rev. D* **65**, 052002 (2002).
- [90] M. A. Shifman, *Z. Phys. C* **4**, 345 (1980).
- [91] K. Gottfried, *Phys. Rev. Lett.* **40**, 598 (1978).
- [92] M. B. Voloshin and V. I. Zakharov, *Phys. Rev. Lett.* **45**, 688 (1980).
- [93] V. A. Novikov and M. A. Shifman, *Z. Phys. C* **8**, 43 (1981).
- [94] R. J. Crewther, *Phys. Rev. Lett.* **28**, 1421 (1972).

- [95] M. S. Chanowitz and J. Ellis, *Phys. Lett.* B **40**, 397 (1972); *Phys. Rev.* D 2490 (1973).
- [96] D. L. Freedman, I. J. Muzinich and E. J. Weinberg, *Ann. Phys. (N.Y.)* **87**, 95 (1974).
- [97] J. S. Collins, L. Duncan, and S. D. Joglekar, *Phys. Rev.* D **16**, 438 (1977).
- [98] D. J. Gross, S. B. Treiman, and F. Wilczek, *Phys. Rev.* D **19**, 2188 (1979).
- [99] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys.* B **165**, 55 (1980).
- [100] M. B. Voloshin, *Phys. Lett.* B **562**, 68 (2003).
- [101] M. B. Voloshin and A. D. Dolgov, *Sov. J. Nucl. Phys.* **35**, 120 (1982) [*Yad. Fiz.* **35**, 213 (1982)].
- [102] M. B. Voloshin, *Sov. J. Nucl. Phys.* **44**, 478 (1986) [*Yad. Fiz.* **44**, 738 (1986)].
- [103] R. S. Chivukula, A. G. Cohen, H. Georgi and A. V. Manohar, *Phys. Lett.* B **222**, 258 (1989).
- [104] R. S. Chivukula, A. G. Cohen, H. Georgi, B. Grinstein and A. V. Manohar, *Ann. Phys. (N.Y.)* **192**, 93 (1989).
- [105] E.V. Shuryak and A.I. Vainshtein, *Nucl. Phys.* B **199**, 451 (1982); *Nucl. Phys.* B **201**, 141 (1982).
- [106] L. S. Brown and R. N. Cahn, *Phys. Rev. Lett.* **35**, 1 (1975).
- [107] M. B. Voloshin, *JETP Lett.* **21**, 347 (1975) [*Pisma Zh. Eksp. Teor. Fiz.* **21**, 733 (1975)].
- [108] M. B. Voloshin, *Phys. Rev.* D **74**, 054022 (2006)
- [109] J. Z. Bai *et al.* [BES Collaboration], *Phys. Rev.* D **62**, 032002 (2000).
- [110] N. E. Adam *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **94**, 232002 (2005)
- [111] M. B. Voloshin and Y. M. Zaitsev, *Sov.Phys.Usp.* **30**, 553 (1987) [*Usp.Fiz.Nauk* **152**, 361 (1987)].
- [112] S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966).
- [113] H. Albrecht *et al.* [ARGUS Collaboration], *Z. Phys. C* **35**, 283 (1987).
- [114] S. Dubynskiy and M. B. Voloshin, arXiv:0707.1272 [hep-ph].
- [115] D. Kreinick [for CLEO Collaboration], Talk at the CHARM 2007 Workshop, Aug.5 - 8, Cornell Univ. URL: http://www.lepp.cornell.edu/charm07/talks/Monday/1055_krenick.pdf
- [116] D. H. Miller *et al.* [CLEO Collaboration], arXiv:0707.1810 [hep-ex].
- [117] A. Lopez *et al.* [CLEO Collaboration], arXiv:0707.1601 [hep-ex].
- [118] B. L. Ioffe and M. A. Shifman, *Phys. Lett.* B **95**, 99 (1980).
- [119] J. Gasser and H. Leutwyler, *Phys.Rept.* **87**, 77 (1982).
- [120] S. Weinberg, in *A Festschrift for I.I. Rabi*, Ed. by L. Motz, Trans, NY Acad.Sci., Ser.II, **38**, 185 (1977).
- [121] Y.-P. Kuang, S. F. Tuan, and T.-M. Yan, *Phys. Rev.* D **37**, 1210 (1988).
- [122] M. B. Voloshin, *Sov.J.Nucl.Phys.* **43**, 1011 (1986) [*Yad.Fiz.* **43**, 1571 (1986)].
- [123] G. Bhanot and M. E. Peskin, *Nucl. Phys.* B **156**, 391 (1979).
- [124] M. B. Voloshin, *Mod. Phys. Lett.* A **19**, 665 (2004).
- [125] M. E. Peskin, *Nucl. Phys.* B **156**, 365 (1979).
- [126] D. Kharzeev and H. Satz, *Phys. Lett.* B **334**, 155 (1994).
- [127] A. B. Kaidalov and P. E. Volkovitsky, *Phys. Rev. Lett.* **69**, 3155 (1992).
- [128] S. J. Brodsky, I. Schmidt, and G. F. de Terramond, *Phys. Rev. Lett.* **64**, 1011 (1990).
- [129] J. Hüfner *et.al.*, *Phys. Rev.* D **62**, 094022 (2000).
- [130] A. Sibirtsev, K. Tsushima, and A. W. Thomas, *Phys.Rev. C* **63**, 044906 (2001).
- [131] R. Vogt, *Phys.Rept.* **310**, 197 (1999).
- [132] T. Barnes, *Eur.Phys.J. A* **18**, 531 (2003).

- [133] T. Song and S. H. Lee, *Phys. Rev. D* **72**, 034002 (2005).
- [134] F. Klingl, S. s. Kim, S. H. Lee, P. Morath and W. Weise, *Phys. Rev. Lett.* **82**, 3396 (1999) [Erratum-*ibid.* **83**, 4224 (1999)] [arXiv:nucl-th/9811070].
- [135] A. Hayashigaki, *Prog.Theor.Phys.* **101**, 923 (1999).
- [136] S. Kim and S. H. Lee, *Nucl.Phys. A* **679**, 517 (2001).
- [137] K. Tsushima, A. Sibirtsev, K. Saito, A. W. Thomas and D. H. Lu, *Nucl.Phys. A* **680**, 280 (2001).
- [138] A. Sibirtsev and M. B. Voloshin, *Phys. Rev. D* **71**, 076005 (2005).
- [139] D. A. Wasson, *Phys. Rev. Lett.* **67**, 2237 (1991).
- [140] K. Tsushima, D. H. Lu, A. W. Thomas, K. Saito and R. H. Landau, *Phys.Rev. C* **59**, 2824 (1999).
- [141] A. Sibirtsev, K. Tsushima and A. W. Thomas, *Eur.Phys.J. A* **6**, 351 (1999).
- [142] B. Friman, S. H. Lee and T. Song, *Phys. Lett. B* **548**, 153 (2002)
- [143] K. Tsushima and F. C. Khanna, *Phys. Lett. B* **552**, 138 (2003).
- [144] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm and H. Stocker, *Phys.Rev. C* **69**, 015202 (2004).
- [145] A. Sibirtsev, *Eur.Phys.J. A* **18**, 475 (2003).
- [146] D. Besson *et al.* [CLEO Collaboration], *Phys. Rev. Lett.*, **96**, 092002 (2006).
- [147] S. Dobbs *et al.* [CLEO Collaboration], arXiv:0709.3783 [hep-ex].
- [148] M. Ablikim *et al.*, [BES Collaboration], *Phys. Lett. B* **652**, 238 (2007).
- [149] M. Ablikim *et al.*, [BES Collaboration], *Phys. Rev. Lett.* **97**, 121801 (2006).
- [150] N. E. Adam *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **96**, 082004 (2006).
- [151] J. Z. Bai *et al.* [BES Collaboration], *Phys. Lett. B* **605**, 63 (2005).
- [152] R. A. Briere *et al.* [CLEO Collaboration], *Phys. Rev. D* **74**, 031106 (2006).
- [153] J. L. Rosner, *Ann. Phys. (N.Y.)* **319**, 1 (2005).
- [154] M. B. Voloshin, *Phys. Rev. D* **71**, 114003 (2005).
- [155] M. B. Voloshin and L. B. Okun, *JETP Lett.* **23**, 333 (1976) [*Pisma Zh.Eksp.Teor.Fiz.* **23**, 369 (1976)].
- [156] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. D* **69**, 071101 (2004).
- [157] D. Atwood and W. J. Marciano, *Phys. Rev. D* **41**, 1736 (1990).
- [158] G. P. Lepage, *Phys. Rev. D* **42**, 3251 (1990).
- [159] N. Byers and E. Eichten, *Phys. Rev. D* **42**, 3885 (1990).
- [160] M. B. Voloshin, *Mod.Phys.Lett. A* **18**, 1783 (2003).
- [161] S. Dubynskiy, A. Le Yaouanc, L. Oliver, J. C. Raynal and M. B. Voloshin, *Phys. Rev. D* **75**, 113001 (2007).
- [162] S. K. Choi *et.al.* [Belle Collaboration], *Phys. Rev. Lett.* **91**, 262001 (2003).
- [163] D. Acosta *et.al.* [CDF Collaboration] *Phys. Rev. Lett.* **93**, 072001 (2004).
- [164] V. M. Abazov *et.al* [D0 Collaboration], *Phys. Rev. Lett.* **93**, 162002 (2004).
- [165] B. Aubert *et.al.* [BABAR Collaboration], *Phys. Rev. D* **71**, 071103 (2005).
- [166] C. Cawlfield *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **98**, 092002 (2007)
- [167] J. L. Rosner, *Phys. Rev. D* **70**, 094023 (2004).
- [168] K. Abe *et.al.* [Belle Collaboration], Belle report BELLE-CONF-0541, 2005, arXiv:hep-ex/0505038
- [169] A. Abulencia *et al.* [CDF Collaboration], *Phys. Rev. Lett.* **98**, 132002 (2007).
- [170] G. Gokhroo *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **97**, 162002 (2006).

- [171] B. Aubert *et al.* [BABAR Collaboration], arXiv:0708.1565 [hep-ex].
- [172] K. Abe *et.al.* [Belle Collaboration], Belle report BELLE-CONF-0540, 2005, arXiv:hep-ex/0505037
- [173] B. Aubert *et.al.* [BABAR Collaboration], *Phys. Rev. D* **74**, 071101 (2005).
- [174] B. Aubert *et.al.* [BABAR Collaboration], *Phys. Rev. D* **71**, 031501 (2005).
- [175] F. E. Close and P. R. Page, *Phys. Lett. B* **578**, 119 (2004).
- [176] E. S. Swanson, *Phys. Lett. B* **588**, 189 (2004).
- [177] N. A. Törnqvist, *Phys. Lett. B* **599**, 209 (2004).
- [178] M. B. Voloshin, *Phys. Lett. B* **579**, 316 (2004).
- [179] A. de Rujula, H. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **38**, 317 (1977).
- [180] M. B. Voloshin, *Phys. Lett. B* **604**, 69 (2004).
- [181] E. S. Swanson, *Phys. Lett. B* **588**, 189 (2004).
- [182] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. D* **73**, 011101 (2006).
- [183] E. Braaten and M. Kusunoki, *Phys. Rev. D* **71**, 074005 (2005).
- [184] M. Suzuki, *Phys. Rev. D* **72**, 114013 (2005).
- [185] M. B. Voloshin, *Int.J.Mod.Phys. A* **21**, 1239 (2006).
- [186] S. Dubynskiy and M. B. Voloshin, *Phys. Rev. D* **74**, 094017 (2006).
- [187] B. W. Lang, arXiv:0710.0165 [hep-ex].
- [188] D. Cronin-Hennessy, *et. al.* [CLEO Collaboration], arXiv:0801.341 [hep-ex].
- [189] M. B. Voloshin, *eConf C060409*, 014 (2006) [arXiv:hep-ph/0605063].
- [190] P. Colangelo, F. De Fazio and S. Nicotri, *Phys. Lett. B* **650**, 166 (2007).
- [191] C. Meng and K. T. Chao, *Phys. Rev. D* **75**, 114002 (2007).
- [192] S. Dubynskiy and M. B. Voloshin, arXiv:0709.4474 [hep-ph].
- [193] C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, *Phys. Rev. D* **76**, 034007 (2007)
- [194] D. V. Bugg, *Phys. Lett. B* **598**, 8 (2004).
- [195] Yu. S. Kalashnikova, *Phys. Rev. D* **72**, 034010 (2005).
- [196] M. B. Voloshin, *Phys. Rev. D* **76**, 014007 (2007).
- [197] L. D. Landau and E. M. Lifshits, *Quantum Mechanics (Non-relativistic Theory)*, Third Edition, Pergamon, Oxford, 1977.
- [198] E. Braaten and M. Lu, arXiv:0710.5482 [hep-ph].
- [199] E. Braaten and M. Lu, arXiv:0709.2697 [hep-ph].
- [200] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, *Phys. Lett. B* **72**, 53 (1977).
- [201] C. Z. Yuan *et al.* [Belle Collaboration], arXiv:0707.2541 [hep-ex].
- [202] S. Dubynskiy and M. B. Voloshin, *Mod.Phys.Lett. A* **21**, 2779 (2006) [arXiv:hep-ph/0608179].
- [203] B. W. Lang, *Measurement of the Total Charm Cross Section by e^+e^- Annihilation at Energies Between 3.97-4.26 GeV*, Ph.D. Thesis, Univ. of Minnesota, October 2006.
- [204] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. Lett.* **95**, 142001 (2005).
- [205] K. Abe *et al.* [Belle Collaboration], arXiv:hep-ex/0612006.
- [206] Q. He *et al.* [CLEO Collaboration], *Phys. Rev. D* **74**, 091104 (2006).
- [207] T. E. Coan *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **96**, 162003 (2006).
- [208] B. Aubert *et al.* [BABAR Collaboration], arXiv:0710.1371 [hep-ex].

- [209] F. J. Llanes-Estrada, *Phys. Rev. D* **72**, 031503 (2005).
- [210] S. L. Zhu, *Phys. Lett. B* **625**, 212 (2005).
- [211] F. E. Close and P. R. Page, *Phys. Lett. B* **628**, 215 (2005).
- [212] E. Kou and O. Pene, *Phys. Lett. B* **631**, 164 (2005).
- [213] L. Maiani, V. Riquer, F. Piccinini and A. D. Polosa, *Phys. Rev. D* **72**, 031502 (2005).
- [214] B. Aubert *et al.* [BABAR Collaboration], *Phys. Rev. Lett.* **98**, 212001 (2007).
- [215] X. L. Wang *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **99**, 142002 (2007).
- [216] K. Abe *et al.* [Belle Collaboration], arXiv:0708.1790 [hep-ex].
- [217] J. L. Rosner, arXiv:0708.3496 [hep-ph].
- [218] D. V. Bugg, arXiv:0709.1254 [hep-ph].
- [219] C. Meng and K. T. Chao, arXiv:0708.4222 [hep-ph].
- [220] X. Liu, Y. R. Liu, W. Z. Deng and S. L. Zhu, arXiv:0711.0494 [hep-ph].
- [221] L. Maiani, A. D. Polosa and V. Riquer, arXiv:0708.3997 [hep-ph].
- [222] S. S. Gershtein, A. K. Likhoded and G. P. Pronko, arXiv:0709.2058 [hep-ph].
- [223] C. F. Qiao, arXiv:0709.4066 [hep-ph].